An inverse approach for material parameter identification in a cyclic bending test using LS-DYNA and LS-OPT

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Summary:

The residual stresses in the blank after forming are the main cause for the subsequent springback in a sheet forming operation. The accuracy of the predicted springback in a Finite Element simulation of the forming operation is very much determined by the quality of the material modeling. Those parts of the workpiece, which in particular contribute to the global springback, have usually been subjected to a bending/unbending deformation mode, when the sheet material has slipped over a tool radius. It is thus of utmost importance that the material model can accurately describe the material response, when it is subjected to such a deformation mode. This is considered by the so-called "hardening law" of the material model. In this context the terms "kinematic" or "mixed" hardening are frequently employed.

There are numerous such hardening models described in the literature. Common for them all is that they involve material parameters, which have to be determined from some kind of cyclic test. In theory, the most simple and straight forward test is a tensile/compression test of a sheet strip. In practice, however, such a test is very difficult to perform, due to the tendency of the strip to buckle in compression. In spite of these difficulties some successful attempts to perform cyclic tension/compression tests have been reported in the literature. However, common for these tests has been that rather complicated test rigs have been designed and used in the experiments, in order to prevent the sheet strip from buckling.

Another method that frequently has been used for the determination of material hardening parameters is the three-point bending test. The advantage of this test is that it is simple to perform, and standard test equipments can be used. The disadvantage is that the material parameters have to be determined by some kind of inverse approach. The current authors have previously, successfully been utilizing this method. In the test the applied force and the corresponding displacement are recorded. The test has then been simulated by means of the Finite Element code LS-DYNA, and the material parameters have been determined by finding a best fit to the experimental force-displacement curve by means of the optimization code LS-OPT, based on a Response Surface Methodology. A problem is, however, that such simulations can be quite time consuming, since the same Finite Element model has to be analyzed numerous times.

In the current paper an alternative numerical methodology is described, in which the Finite Element problem only has to be solved a limited number of times, and, thus, considerably reducing the computational cost. In this new methodology a computed moment-curvature curve is fitted to an experimental one. A complicating factor is, however, that all information to determine a moment-curvature relation is not available directly from the experiment. Therefore, the problem has to be solved in two nested iteration loops, where the optimization loop is contained within an outer loop, in which the FE-analysis is performed. It is demonstrated that the parameters determined by this new method correspond excellent to the ones determined by the conventional method.

Keywords:

Bending test, Cyclic bending, Inverse identification, Kinematic hardening

1   Introduction

The material parameters that are included in various existing hardening laws have to be determined from an experiment that represents a loading-reloading situation, i.e. some kind of cyclic test. In theory, the most simple and straight forward test is a tensile/compression test of a sheet strip. In practice, however, such a test is very difficult to perform, due to the tendency of the strip to buckle in compression. In spite of these difficulties some successful attempts to perform cyclic tension/compression tests have been reported in the literature, i.e. [1]-[4]. However, common for these tests has been that rather complicated test rigs have been designed and used in the experiments, in order to prevent the sheet strip from buckling.

Another method that frequently has been used for the determination of material hardening parameters is some kind of bending test [5]-**Fehler! Verweisquelle konnte nicht gefunden werden.**. In the present study a cyclic three point bending test is used. The advantage of this test is that it is simple to perform, and standard test equipment can be used. The disadvantage is that the material parameters have to be determined by some kind of inverse approach. The test has been suggested in the literature by e.g. Zhao and Lee [5]. The equipment used in the current experiments has previously been described in Omerspahic et al. [6].

In the present investigation four different kinematic hardening laws of various complexities were studied. Furthermore, the responses of four different steel grades were investigated, of which two are accounted for herein. The three point bending experiments were simulated by the Finite Element code LS-DYNA [9]. The optimization of the material parameters were governed by a successive Response Surface Methodology implemented in the optimization tool LS-OPT [10].

2   Kinematic hardening laws

The four different kinematic hardening laws being part of the current investigation are very briefly described below. A more detailed description is found in Eggertsen and Mattiasson [11].

2.1   Mixed hardening

The hardening law that is called "mixed hardening" in this study is a combination of isotropic and kinematic hardening, where the proportion of isotropic and kinematic hardening is weighted with a scalar m. The scalar m represents the ratio of plastic strain associated to isotropic hardening, whereas the ratio $(1-m)$ is left for the kinematic hardening response. From this it follows that:

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\boldsymbol{\varepsilon}}^p_{iso} + \dot{\boldsymbol{\varepsilon}}^p_{kin} = m\dot{\boldsymbol{\varepsilon}}^p + (1-m)\dot{\boldsymbol{\varepsilon}}^p$$
$$0 \leq m \leq 1 \tag{1}$$

The evolution of the back-stress $\boldsymbol{\alpha}$ can then be expressed as:

$$\dot{\boldsymbol{\alpha}} = \left(H' - \tilde{H}'m \right)\frac{\dot{\bar{\varepsilon}}^p}{\bar{\sigma}}\left(\boldsymbol{\sigma} - \boldsymbol{\alpha} \right) \tag{2}$$

where now H' is the current plastic slope (at $\bar{\varepsilon}^p$), and $\tilde{H}'$ is the plastic slope related to the isotropic hardening.

The mixed hardening law is able to consider the Bauschinger effect and the permanent softening behavior. Compared to the isotropic hardening law there are, thus, one additional material parameter, m, and three additional history variables, the three components of $\boldsymbol{\alpha}$.

2.2   Armstrong-Frederick hardening

The second hardening law is the law by Armstrong and Frederick, which states that the back-stress evolution is given by:

$$\dot{\boldsymbol{\alpha}} = C_x \left[\alpha_{sat}\frac{\boldsymbol{\sigma} - \boldsymbol{\alpha}}{\bar{\sigma}} - \boldsymbol{\alpha} \right]\dot{\bar{\varepsilon}}^p \tag{3}$$

where α_{sat} and C_x are material parameters. There are, thus, two additional material parameters and three additional history variables. The Armstrong-Frederick hardening law is able to consider the Bauschinger effect and the transient behavior.

2.3   Geng-Wagoner hardening

The Geng-Wagoner hardening law is an extension of the Armstrong-Frederick law, and involves two surfaces in stress space: A yield surface and a bounding surface. The Geng-Wagoner law includes translation of the bounding surface, in order to capture the permanent softening effect. The evolution equations are

$$\dot{\boldsymbol{\alpha}} = C_x \left[\frac{\alpha_{sat}}{\overline{\sigma}} (\boldsymbol{\sigma} - \boldsymbol{\alpha}) - (\boldsymbol{\alpha} - \boldsymbol{\beta}) \right] \dot{\overline{\varepsilon}}^p \tag{4}$$

$$\dot{\boldsymbol{\beta}} = \frac{H' - \tilde{H}' m}{\overline{\sigma}} (\boldsymbol{\sigma} - \boldsymbol{\alpha}) \, \dot{\overline{\varepsilon}}^p \tag{5}$$

where $\boldsymbol{\alpha}$ is the center of the yield surface and $\boldsymbol{\beta}$ is the center of the bounding surface. As can be seen from Eq. (4), the hardening of the bounding surface is governed by the mixed hardening law (Eq. (3)).

2.4   Yoshida-Uemori hardening

The final hardening law is the one by Yoshida and Uemori. The model includes both translation and expansion of the bounding surface, while the active yield surface only evolves kinematically. The evolution of the back-stress is expressed as:

$$\dot{\boldsymbol{\alpha}} = \dot{\boldsymbol{\alpha}}^* + \dot{\boldsymbol{\beta}} \tag{6}$$

with

$$\dot{\boldsymbol{\alpha}}^* = C_x \cdot \left(\frac{B + R - Y}{Y} \cdot (\boldsymbol{\sigma} - \boldsymbol{\alpha}) - \boldsymbol{\alpha}^* \right) \cdot \dot{\overline{\varepsilon}}_p$$

$$\dot{\boldsymbol{\beta}} = k \cdot \left(\frac{b}{B + R} \cdot (\boldsymbol{\sigma} - \boldsymbol{\beta}) - \boldsymbol{\beta} \right) \cdot \dot{\overline{\varepsilon}}_p \tag{7}$$

where $\boldsymbol{\alpha}^*$ is the relative kinematic motion of the yield surface with respect to the bounding surface, $\boldsymbol{\beta}$ is the centre of the bounding surface, B is the initial size of the bounding surface, Y is the size of the yield surface, C_x and k are material parameters, and R is the isotropic hardening of the bounding surface:

$$\dot{R}(\overline{\varepsilon}^p) = k \, (R_{sat} - R(\overline{\varepsilon}^p)) \dot{\overline{\varepsilon}}^p \tag{8}$$

In Eq. (8) R_{sat} is a material parameter describing the upper limit of isotropic hardening. The version of the Yoshida-Uemori model that is used in this work is a modified version that includes four material parameters, C_x, k, b and h.

3   Experimental set-up

The test set-up is illustrated in Figs. 1 and 2. Especially the design of the end supports should be noticed. These provide a moment free support, while the sheet strip is allowed to slip freely, between two rollers, in the axial direction. The punch in the middle is moved with a prescribed sinusoidal displacement. The distance between the end supports is 100 mm, and the width of the sheet strip is 20 or 25 mm depending on the materials. The punch force was measured by means of a load cell. During the test the punch force and punch displacement were recorded. The error in displacement reading is estimated to ± 0.02mm and in force reading ± 3N.
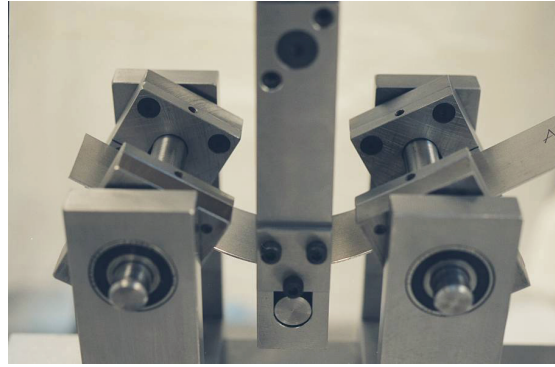
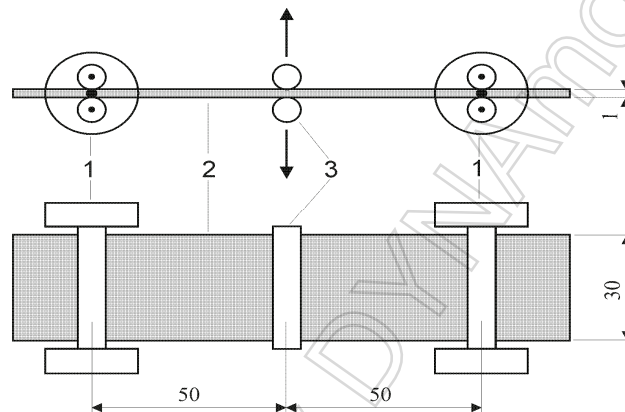*Figure 1 : A photo of the experimental set-up used in the three point cyclic bending tests*

Figure 2 : A sketch of the test arrangement

4   Finite Element model

In the inverse approach for determining the hardening parameters, the bending test have been simulated by means of the explicit FE-code LS-DYNA [9]. One quarter of the sheet strip has been modeled with triangular shell elements. Convergent results have been obtained with 160 elements and nine integration points through the thickness. The FE-model is shown in Figure 3. Special care has been devoted to a correct modeling of the end supports. Just as for the experimental set-up, the support is free to rotate around its own axis. The supports are modeled with two rigid plates with a frictionless contact to the specimen, such that the sheet material can slip freely in the longitudinal direction.
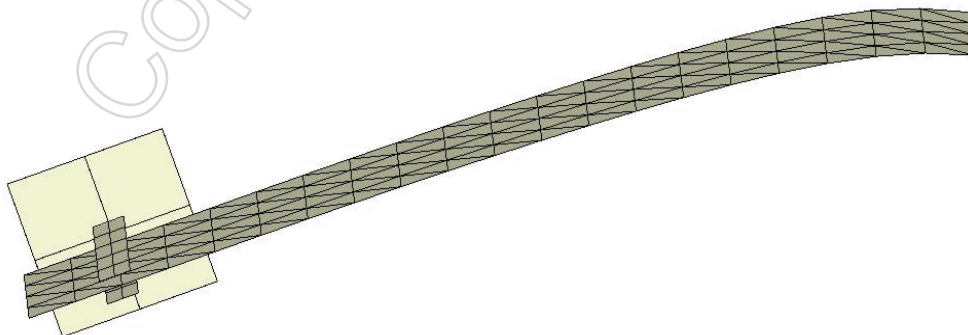

Figure 3 : FE-model of the three point bending test

5 Optimization algorithm: Response Surface Methodology

In the Optimization procedure used to determine the material hardening parameters a Response Surface Methodology (RSM) was used. The optimization software was LS-OPT [10]. RSM is a mathematical method for constructing smooth approximations of complex models in a design space. The approximations are calculated at numerous experimental points in the design-space. In other words, the experimental points construct an approximate design space wherein an objective function can be optimized. The number of necessary experimental points is decided by the number of design parameters, the choice of polynomial approximation and an experimental design criterion, deciding where the experimental points are placed in the design space. In this work the D-optimality design criterion is used, basically because it is the default in the LS-OPT software. The number of experimental design points for this criterion is listed in Table 1. In Figure 4 a response surface with two variables is illustrated schematically.
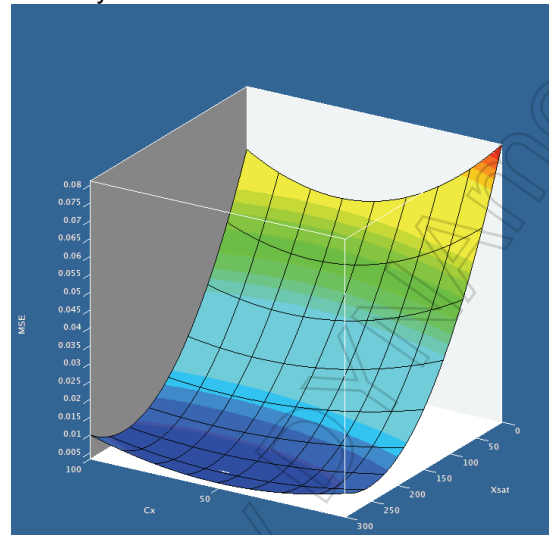

Figure 4 : Example of a response surface with two variables and quadratic approximation of the design space

Table 1*. Number of experimental points required for experimental designs for linear and quadratic approximation, respectively.*

Number of variables	Linear approx.	Quadratic approx.
1	4	5
2	5	10
3	7	16
4	8	23
5	10	32

6 Basic identification procedure

The unknown hardening material parameters in the hardening laws were first evaluated by comparing simulated and experimental results and compute the mean squared error (MSE) between the computed punch force based on a prescribed displacement, and the punch force measured in the experimental three point bending test. In this work the MSE is defined as:

$$\varepsilon(\mathbf{p},t) = \frac{1}{P}\sum_{p=1}^{P}\left(f(\mathbf{p},t)-G(t)\right)^2 \qquad (1)$$

where $f(\mathbf{p},t)$ is the predicted punch force at time t, and $G(t)$ is the corresponding value from the experimental three point bending test

Based on an initial guess the LS-OPT software prepare N input decks for FE-simulations, where N is the number of experimental design points needed for building the design space. Each input deck has different material parameter set-ups inside the design space. Each FE-simulation is run and the predicted punch forces are compared to the experimental punch forces by means of the Mean Squared Error, Eq. (1). Based on the MSE values the LS-OPT software calculates an optimum parameter set-up by means of the Response Surface Methodology. If a converged solution is not

found, the process starts all over again, but with a new region of interest in the design space. The advancement of the region is made by a pan (moving the center of region of interest) and zoom (reduce the size of region of interest) method. A schematic illustration of the pan and zoom procedure is shown in Figure 5.
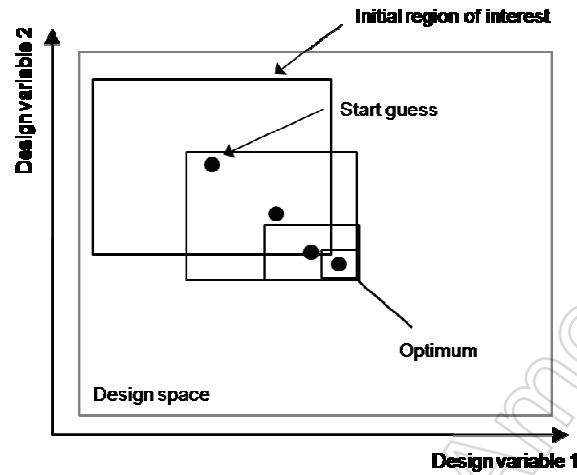

Figure 5 : Schematic illustration of the pan and zoom method for the case of two design variables

For building the design space, quadratic approximations have been used. Therefore, the FE-model presented earlier in Chapter 4 has been calculated between 10 and 23 times in each iteration step for the different hardening models. Obviously, this is a very time and computer consuming procedure, and in order to reduce the computational effort, a modification of this method is presented.

7   Modified identification procedure

As can be seen in Table 1, there is a need for a large number of numerical predictions for the identification of material parameters, if the methodology described in Chapt. 6 is used. In order to reduce the total CPU-time a more effective identification procedure is presented below.

Instead of using the punch force, the bending moment can be used as target values in the MSE criterion. Eq. (1) can then be rewritten as:

$$\varepsilon(\mathbf{p},t) = \frac{1}{P}\sum_{p=1}^{P}\left(M_c(\mathbf{p},t) - M_t(t)\right)^2 \tag{2}$$

where M_c represents the calculated bending moments at time t as a function of the unknown material parameter vector $\mathbf{p}$, and M_t represents target bending moments at time t. The Mean Squared Error function given by Eq. (2) is now considered as the objective function, in the design space defined by:

$$a_i \le p_i \le b_i \qquad i = 1,2,...,n \tag{3}$$

where a_i and b_i are the lower and upper limit respectively and n is the number of unknown material parameters.

The target moment at time t is calculated from the measured punch force P and the calculated normal force F_x (see Fig. 5) as:
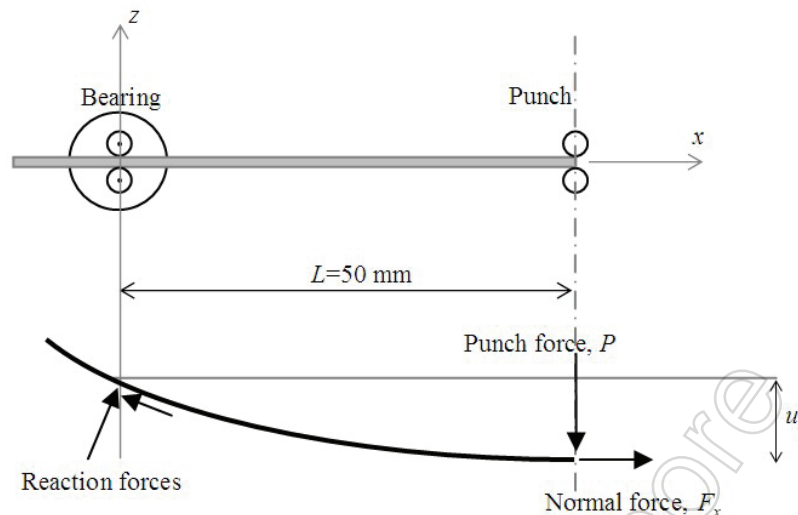
$$M_t^{(t)} = \frac{P}{2} \cdot L - F_x \cdot u_z \tag{4}$$

Figure 6 : Acting forces on one half of the three point bending test

In the calculation of the bending moments, $M_c(p)$, it is assumed that the strains are linearly distributed through the thickness. Based on this assumption the bending moment about the sheet centerline at time t, is determined by integrating the tangential stresses, $\sigma_x^{(t)}(\mathbf{p},z)$, through the thickness such that:

$$M_c^{(t)}(\mathbf{p}) = w \cdot \int_{-t/2}^{+t/2} \sigma_x^{(t)}(\mathbf{p},z) \cdot z\,dz \tag{5}$$

where t and w are the sheet thickness and sheet width, respectively, and z represents the through thickness coordinate. However, the calculation of the tangential stresses presupposes that the strains at the extreme layers are known. If the strains are not known the approach has to be slightly adjusted. The adjustment is based on convergence of the strains at extreme layers, and can be described as follow:

- Based on a start guess the strains at the extreme layers, the normal force F_x and the z-displacement, u_z, of the sheet under the punch are calculated by a FE-simulation. In this work LS-DYNA is used to perform the calculations.
- From these results the unknown material parameters are determined by means of Response Surface Optimization and the object function described by Eq. (5). LS-OPT is used to solve the response surface problem, and the optimization procedure follows the same scheme as for the basic model. However, the FE-simulations in the first step are replaced by a simple material routine, calculating the stresses based on the given strains. The obtained material parameters are then used in a new FE-simulation. This procedure is repeated until convergence of the extreme layer strains is obtained.

8   Performances of the models

The performances of the two models were evaluated based on two criteria: the MSE-value and the total CPU-time. The MSE-value is an indicator of the accuracy of the material model with the obtained material parameters (the lower MSE-value, the better accuracy).

It should be noted that the CPU-time is strongly dependent on the starting guess, and the values presented in this report are based on an arbitrary but reasonable initial guess. Furthermore, it should be mentioned that the CPU-times indicated in the tables below, is the time to run the problem on one, single processor. For the basic identification procedure, there is a possibility to perform the calculations on several parallel processors. This means that the real time (clock time) for the identification procedure can be reduced substantially.

The CPU-times and the MSE:s for the two methods and for two materials (TKS-DP600 and TKS-220IF) are presented in Tables 2 and 3. The tables clearly show that the two methods yield more or less the same results in terms of MSE-value. However, the total CPU-time is much lower for the modified method.

Table 2. Obtained MSE and CPU-time for the modified identification procedure for the TKS-DP600 material.

	Basic identification method		Modified identification method	
	MSE	CPU-Time [h]	*MSE*	CPU-Time [h]
Isotropic hardening	0.34562		0.34562	
Mixed hardening	0.09105	6.0	0.09096	2.0
Armstrong-Frederick	0.00463	16.8	0.00456	2.8
Geng-Wagoner	0.00213	26.4	0.00219	4.0
Yoshida-Uemori	0.00168	75.0	0.00171	8.0

Table 3. Obtained MSE and CPU-time for the modified identification procedure for the TKS-220IF material.

	Basic identification method		Modified identification method	
	MSE	CPU-Time [h]	*MSE*	CPU-Time [h]
Isotropic hardening	0.43772		0.43772	
Mixed hardening	0.09853	8.0	0.09804	2.2
Armstrong-Frederick	0.08605	18.6	0.08621	3.0
Geng-Wagoner	0.07653	23.2	0.07598	4.3
Yoshida-Uemori	0.07186	82.5	0.07164	9.0

9   Conclusions

The cyclic tree point bending test has been used to determine the material parameters for various hardening laws. An inverse approach has been used, in which the test has been simulated by means of the FE-code LS-DYNA. A Response Surface Method and the optimization code LS-OPT have been used to determine the optimal material parameters.
In the original methodology the parameters were determined to give an optimal fit to the measured force-displacement curve. This procedure is, however, quite time consuming, since numerous FE-simulations have to be performed.
In the current paper an alternative procedure is proposed, in which the calculated moment-curvature curve is fitted to the corresponding measured one. It has been demonstrated that this procedure offers considerable advantages in terms of computing time.

10   Acknowledgements

11   Literature

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