Transmission loss simulation of acoustic elements in LS-DYNA®

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1 Introduction

This paper presents validated simulations of transmission loss in LS-DYNA for basic acoustic elements in exhaust systems.

According to [1] there are several indicators available that describe the acoustic performance of an exhaust system and its components. These mainly include transmission loss (TL), insertion loss (IL) and noise reduction (NR). The TL is a ratio of sound power level between the inlet wave entering and the transmitted wave exiting the element. Acoustic element termination has to be anechoic, since the TL is a property of the acoustic element only. The NR is sound pressure level difference across the element. The IL is the loss of sound power from the insertion of an arbitrary acoustic element. In this paper we will focus on the TL only. There are several applicable methods in use to measure the TL. The most common and popular approach for measuring transmission loss is decomposition method or sometimes called “three-pole method”. The method is based on the decomposition theory. The basic idea of the method is that the sound pressure may be decomposed in its incident and reflected waves. When the pressure wave is decomposed, the TL can be calculated.

Sound is one of the key trigger elements that make customer want to buy an Akrapovič exhaust system. Well-done sound solutions have been so far developed with the testing of different exhaust system prototype configurations. This demands time and costs to build and test every new configuration. Therefore, we had been always looking for faster solutions. The goal in our company was to create measurement-validated probational simulation models in LS-DYNA to examine the acoustic performance of such parts.

Acoustic component performance prediction is a good example of the use of simulation software in industrial applications. LS-DYNA is one of the widely used finite element codes for solving complex mechanical problems. One of the recent developments is the addition of a vibro-acoustic solver, which enables users to perform a number of vibro-acoustic analyses in the frequency domain. In order to obtain a numerical solution in our case we have used the recently implemented *FREQUENCY_DOMAIN_ACOUSTIC_BEM keyword in LS-DYNA. This new keyword allows users to run acoustic computations based on boundary element method (BEM).

2 Theoretical background

2.1 Acoustics theory basis

Many engineers require reliable simulations of processes in which acoustic waves encounter obstacles. Unless the geometry of the incident obstacle is particularly simple, the analytical solution is usually impossible, and consequently numerical solutions are required. The governing acoustic equation for propagation of sound waves in homogeneous medium at rest [2] is

$$\Delta P - \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} = 0 \qquad (1.1)$$

where c is the speed of sound and

$$\Delta = \nabla^2$$

is an abbreviation for the Laplacian operator. We call P the real acoustic pressure which is given by

$$P(\mathbf{x},t) = \Re\left(u(x)e^{-i\omega t}\right), \tag{1.2}$$

where $\Re$ denotes the real part of function u, that we call complex acoustic pressure and which is complex valued in general. Angular frequency is denoted as $\omega = 2\pi f$ and f is the frequency measured in Hz and $i = \sqrt{-1}$. Substituting (1.2) into (1.1), we observe that u satisfies the Helmholtz equation

$$\Delta u + k^2 u = 0, \quad in\, D \subset R^d, \quad d = 1,2,3, \tag{1.3}$$

where the positive constant k is the wave number given as $k = \omega / c$.
The most common and physically relevant boundary condition to satisfy (1.3) is the impedance boundary condition

$$\frac{\partial u}{\partial \mathbf{n}} + ik\beta u = g, \quad on\, \partial D, \tag{1.4}$$

where ∂D is the boundary of a D. In this equation, the first article denotes the rate of increase in u in the direction $\mathbf{n}$, where $\mathbf{n}(\mathbf{x})$ represents the unit normal at boundary. The function g on the right hand side of the equation is zero for problems where we are given an incident wave and a stationary reflection and have to compute resulting acoustic field. Thus, for vibro-acoustic problems, the boundary condition is given as

$$\frac{\partial u}{\partial \mathbf{n}} = -ik\beta u = -i\rho\omega v_n \tag{1.5}$$

where $\mathbf{n}$ is pointing away from the acoustic volume (fluid in our case), ρ is fluid density and v_n is the normal velocity at a boundary [3].

2.2 Boundary element method

Thus, we can directly transform equation (1.3) into a boundary integral equation with the use of Green's theorem. Consequently the pressure at any boundary point of the fluid medium can be expressed as an integral of the sum of pressure and velocity over a surface [2], [4], given by the equation

$$C(\mathbf{x}) \cdot u(\mathbf{x}) = \int_{\partial D} \left[G(\mathbf{y},\mathbf{x}) \frac{\partial u(\mathbf{y})}{\partial \mathbf{n}_y} - u(\mathbf{y}) \frac{\partial G(\mathbf{y},\mathbf{x})}{\partial \mathbf{n}_y} \right] ds(\mathbf{y}), \quad x \in \partial D \tag{1.6}$$

where $G(\mathbf{y}, x)$ is the Green's function, $\mathbf{n}_y$ is the normal on the surface $s(\mathbf{y})$ and $C(x)$ is the jump term resulting from the equation derivation. At all points on ∂D except for edges and corners it holds that

$$C(\mathbf{x}) = \frac{1}{2}.$$

The BEM is obtained by putting FEM philosophy of discretization to a boundary integral equation formulation of the acoustic problem. We start by dividing the boundary ∂D into N small boundary elements. We assume that the boundary integral equation (1.6) can be approximated by the BEM. Approximate solution, usually made by a simple polynominal function in each element, of the integral equation is the pressure u on the boundary ∂D. Since we have a set of elements, we get a system of equations. One of the simplest methods for determining the values u_j from a set of equations is the *collocation method*. LS-DYNA defines the equation for each node at the boundary [4]. Typically we pick a point $\boldsymbol{x}_i$ in the centroid of each element and require the equation (1.6) to hold exactly at each of these *collocation points*. This gives a set of N linear equations to determine N unknowns. LS-DYNA solver takes care of this part.

Knowing for pressure or velocity on the surface allows for calculation of pressure at any of the field points $\boldsymbol{x}_i$. Most frequently only the half of the variables are known on the surface domain (various combinations of pressure and velocity). It is wise to note that BEM linear system depends on the frequency via Green's function. Consequently, the system of linear equations has to be solved separately for each frequency.

2.3 Decomposition method and TL

There are several applicable methods in use to measure the TL. The most common and popular approach for measuring transmission loss is decomposition method or sometimes called "three-pole method". The method is based on the decomposition theory. The basic idea of a method is that any sound pressure may be decomposed in its incident and reflected waves. When the pressure wave is decomposed, the TL can be calculated.

According to [1] the TL is a ratio of acoustical power level between the inlet wave entering and the transmitted wave exiting the element under the assumption of anechoic termination (equation 1.7).

$$TL = 10\log_{10}\frac{W_i}{W_t}, \tag{1.7}$$

where W_i is the incident sound power and W_t is the transmitted sound power. We know that the sound power W is related to pressure p via the equation

$$W = \frac{p^2}{\rho c}S. \tag{1.8}$$

In equation (1.8) ρ is the density of a fluid, c is the speed of sound and S is the square section of inlet/outlet. Substituting sound power level from the equation (1.8) into equation (1.7), the TL can be expressed as

$$TL = 20\log_{10}\frac{p_i}{p_t} + 10\log_{10}\frac{S_i}{S_t}. \tag{1.9}$$

Hence, in the three-pole method sound power can be measured by simply measuring the sound pressure level at inlet and outlet [5]. Because of the anechoic termination the acoustic wave in the outlet tube contains only an outgoing wave. On the contrary, the acoustic wave in the inlet tube contains an input wave as well as the reflected wave. To extract the incoming wave from the incident wave two measurement points have to be defined. Let x_1 and x_2 be the coordinates of the two points (microphones) along the length of an acoustic component (Fig. 1). The corresponding sound pressures p_1 and p_2 at these two points are given in a complex domain as

$$p_1 = p_i\, e^{-ikx_1} + p_r\, e^{-ikx_1}, \qquad p_2 = p_i\, e^{-ikx_2} + p_r\, e^{-ikx_2}, \tag{2.0a, b}$$

where p_i stands for the pressure of the incoming wave, and p_r represents the reflected wave. We used the FFT to transform real pressure data from the measurements to the complex domain. Solving the equations (2.0a) and (2.0b) for p_i we obtain

$$p_i = \frac{1}{2i\sin[k(x_2 - x_1)]}\left(p_1\, e^{ikx_2} + p_2\, e^{ikx_1}\right). \tag{2.1}$$

Third point where we measure p_3 can be placed anywhere in the outlet tube. Thus, the TL of the acoustic component can be evaluated by the equation (1.9) as

$$TL = 20\log_{10}\left(\frac{|p_i|}{|p_3|}\right) + 10\log_{10}\frac{S_i}{S_t}. \tag{2.2}$$

This will be the fundamental equation (2.2) for the calculation of TL in this article. Since sections S_i of the inlet and S_t of the outlet is the same for all acoustic configurations in our case, we can neglect the last article in the equation (2.2).

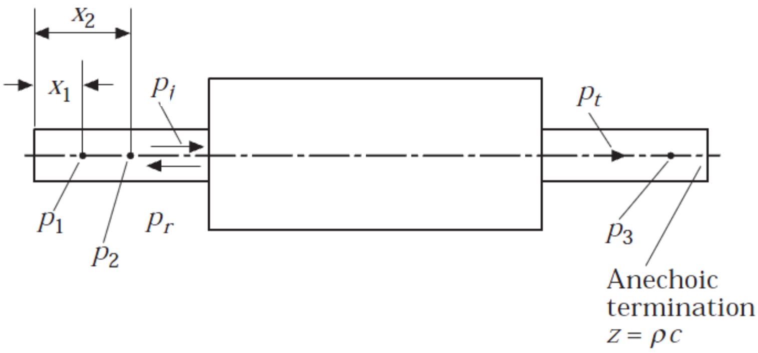

Fig. 1: Setup of decomposition theory.

The major flaw of the three-pole decomposition method is that it requires an anechoic termination to obtain reliable results. That is why four-pole method with an additional microphone at the outlet is widely used to eliminate reflected waves from the transmitted wave.

3 Experimental setup

The entire experimental arrangement is shown in Fig. 2. On the right hand-side is the wooden sound source enclosure with a speaker inside. From the source enclosure sound waves are forced through a tapered conical tube to a round tube with the holes for the first and second microphone. After the inlet microphone setting, an acoustic component with its own impedance and geometry characteristics is placed. Third microphone is mounted after the acoustic component and before the anechoic termination. Anechoic termination consists of three steel tubes filled with damping wool and a tetrahedron foam plug at the far-left end. All three microphones are connected to the data acquisition system which transforms the signal strength into SI unit [Pa] for pressure.

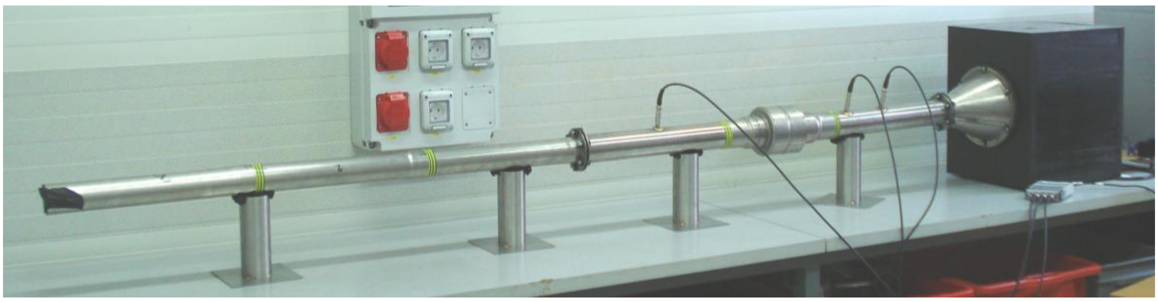

Fig. 2: Experimental setup for three-pole method.

Valid frequency measurement range is limited by impedance tube diameter d, microphone diameter d_m and microphone spacing s [6]. The impedance tube diameter d has an influence on the range of frequencies for which plane wave can occur in a tube. The range of frequencies can be determined via the following equation

$$0 < f < \frac{1.84c}{\pi d}. \tag{2.3}$$

Considering (2.3), the diameter of the impedance tubes ensures that only the plane waves up to 3350 Hz will propagate through the tube. The microphone diameter d_m enables valid pressure measurements to be made up to 9800 Hz, found using

$$f_{up} = \frac{0.2c}{d_m}. \tag{2.4}$$

The valid frequency range due to the microphone spacing s is 31 – 1247 Hz, determined by

$$\frac{0.01c}{s} < f < \frac{0.8c}{2s}. \tag{2.5}$$

Thus, frequency range of 0-2000 Hz was allowed in the measuring process although only the data up to 1000 Hz was used for the analysis. The distance of the first (rightmost) microphone to the acoustic source was chosen to be longer than three impedance tube diameters to avoid measuring non-planar waves. The distance between the third (leftmost) microphone and the damping region in anechoic tube termination is larger than two tube diameters. This way, higher order modes which are created by reflections from the termination tube have a sufficient distance to decay before they reach the third microphone. The TL for all acoustical components was than calculated by the decomposition method.

It has to be emphasized that LS-DYNA simulation results were very helpful at the three-point measurement chain optimization at the very beginning. Before, we had used a simple two-microphone method that gave semi-reliable results which were in spite of that used as one of the key indicators.

4 Simulation model and input parameters determination

All acoustic elements were meshed with shell elements in LS-PrePost®. An example of a simple expansion chamber is presented in Fig. 3. All models comprise inlet, outlet and body segments on which the boundary conditions are applied.

Fig. 3: The round expansion chamber.

The element size should not be more than 1/6 wavelength of the highest frequency [7]. In our case, the upper frequency in the analysis is 1000 Hz. Hence, maximum element size should not be greater than 0.057 m, obtained by

$$\frac{\lambda}{6} = \frac{c}{6f}. \tag{2.6}$$

Elastic stainless steel AISI 304L material properties were used for all acoustic components Tab. 1.

Tab. 1: Stainless steel AISO 304L properties.

Parameter	**Value**
Density	7850 kg/m^3
Elastic modulus	195 GPa
Poisson's ratio	0.29

Properties for air as an acoustic medium are given in Tab. 2.

Tab. 2: Air properties at room temperature.

Parameter	**Value**
Air density	1.21 kg/m^3
Wave propagation speed in medium	343 m/s
Reference pressure	2.0E-5 Pa

We exercised control over the solving termination time through employment of ENDMASS keyword. This keyword stops the job when percentage change in the total mass for termination of calculation exceeds pre-defined value [7].

The boundary element method acoustics analysis in frequency domain can be activated by the keyword *FREQUENCY_DOMAIN_ACOUSTICS_BEM. A uniform complex pressure in the frequency range of 1–1000 Hz is applied at the inlet to excite the whole model. Curve for real part of the pressure is constant and imaginary part is zero. Impedance is prescribed at the outlet in a complex form as well, where real part is constant and imaginary is zero. On segments of the body normal velocity is prescribed through a load curve. Normal velocity is set to zero for all frequencies.

In order to compare the results of simulations with measurements we had to define three exterior field points inside the model. These points were located on a centerline at similar longitudinal distances like in real case (Fig. 1) and were numbered according to their succession.

5 Results and analysis

5.1 Round expansion chamber

One of simple acoustic elements is an axisymmetric expansion chamber (Fig. 4), where pressure waves on their way encounter only bigger cross section and volume. In most cases, chamber volume, length and open section area are important and give characteristic acoustic properties to the component.

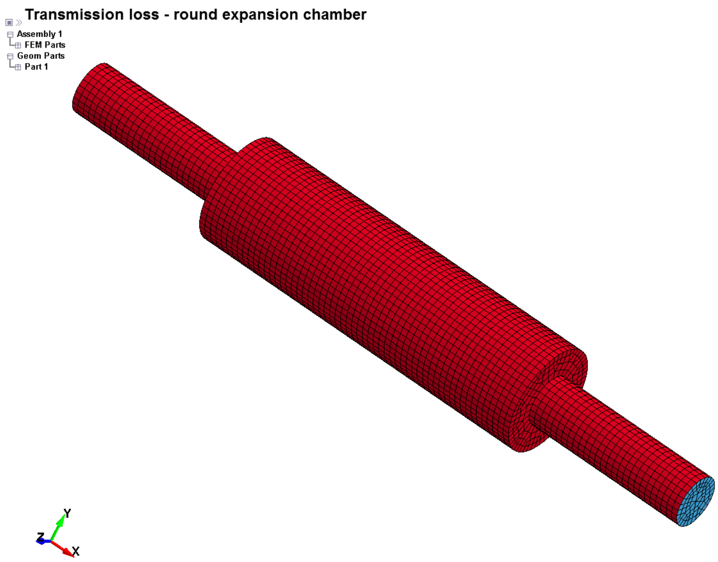

Fig. 4: Meshed model of round expansion chamber.

The transmission loss curves for the range of frequencies 1 Hz to 1000 Hz are plotted in Fig. 4. Simulated and measured (DT) TL curves clearly show that the round expansion chamber is efficient for the tones near 195, 565 and 945 Hz. Measured data validates results from simulation very well. Scatter of the measured data curve is plausibly due to external factors and not the expansion chamber itself. Presence of unwanted noise in the signal is common for three-pole method and can be usually reduced using supplementary microphone and performing measurements in an anechoic room.

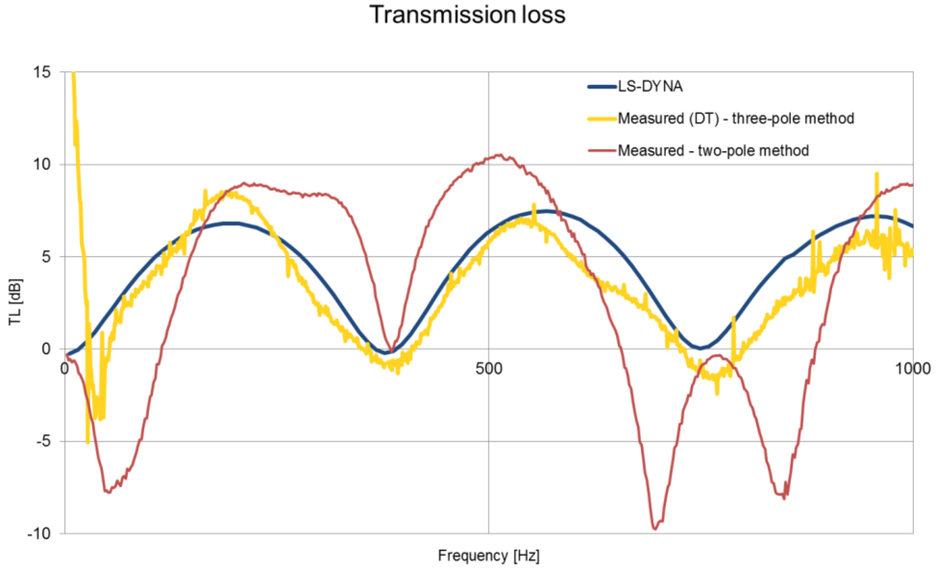

Fig. 4: Comparison between experimental and simulated transmission loss data for an axisymmetric round expansion chamber.

Round expansion chamber has the highest transmission loss of approximately 7 dB at the frequency of 190 Hz, 570 Hz and 960 Hz. Meanwhile the amplitude of simulated TL remains the same throughout the whole frequency range, the peak amplitude of measured TL drops linearly from 8 dB to 6 dB.

Red\dashed line represents the acoustic data measured with the previously used two-microphone technique. We can observe that three-point method gives us much better and reliable results. Further on, only three-pole test method is used to measure acoustic behaviour.

5.2 Helmholtz resonator

Helmholtz resonator is a container of fluid medium with an open hole or neck (Fig. 5). It works due to medium elasticity and energy conservation law. An air in a cavity will exhibit a single resonant frequency in ideal case. When the fluid medium (usually air) is forced into the cavity, the pressure inside it will increase. In the very next instant, external force which is pushing the air into a resonator is removed and consequently higher-pressure medium inside will flow out. Hence, somewhat like a mass on a spring the pressure at the hole in the cavity will oscillate. We can control for the resonant frequency by three determination factors: square section of cavity opening, volume of cavity, and length of opening neck (Fig. 5).

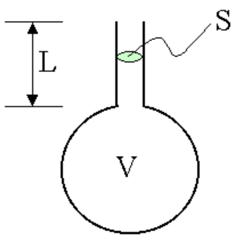

Fig. 5: Simplified Helmholtz resonator model with its characteristic parameters: V – cavity volume, S – neck cross section, L – neck length.

Resonators of different shapes are frequently used in exhaust systems. One of the basic resonator shapes is presented in Fig. 6. An empty container with certain volume is connected to a tube with the neck of certain length and cross-section.

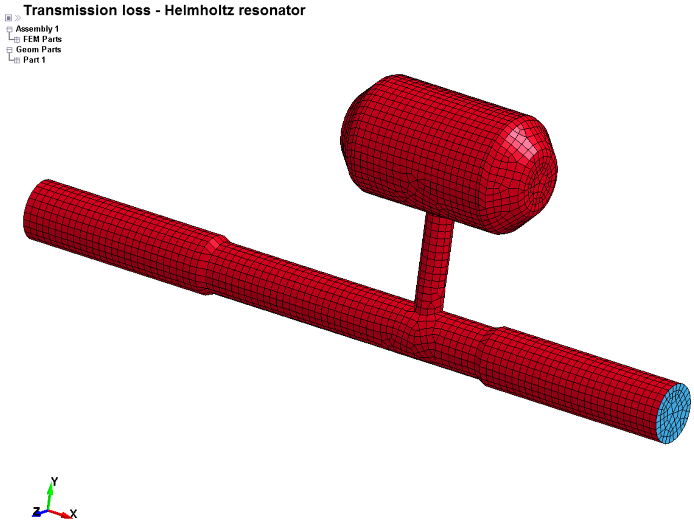

Fig. 6: The simulated Helmholtz resonator assembly.

TL for the range of frequencies 1 Hz to 1000 Hz is plotted in Fig. 7. Again, we can observe very good matching of the measured and simulated data at the peak frequency. Simulated TL amplitude is 17 dB and measured peak amplitude is 27 dB.

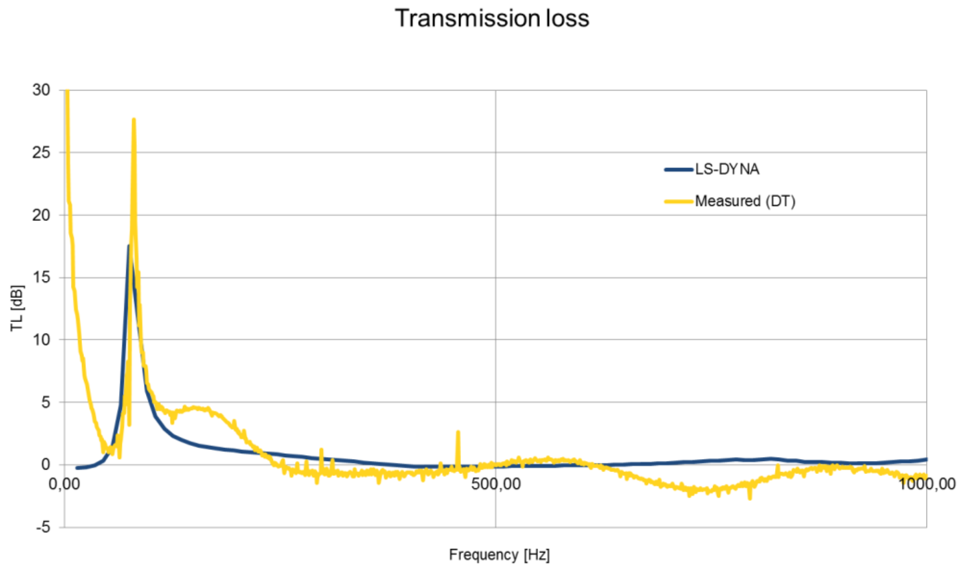

Fig. 7: Comparison between experimental and simulated transmission loss data for a certain configuration of Helmholtz resonator.

In order to see precisely where the peak frequency is located, we increased frequency resolution in the range from 50 to 150 Hz (Fig. 8). As a consequence we get the simulated TL peak amplitude of 37 dB at 77 Hz. The difference in frequencies between the peaks of measured and simulated TL is no more than 2 Hz.

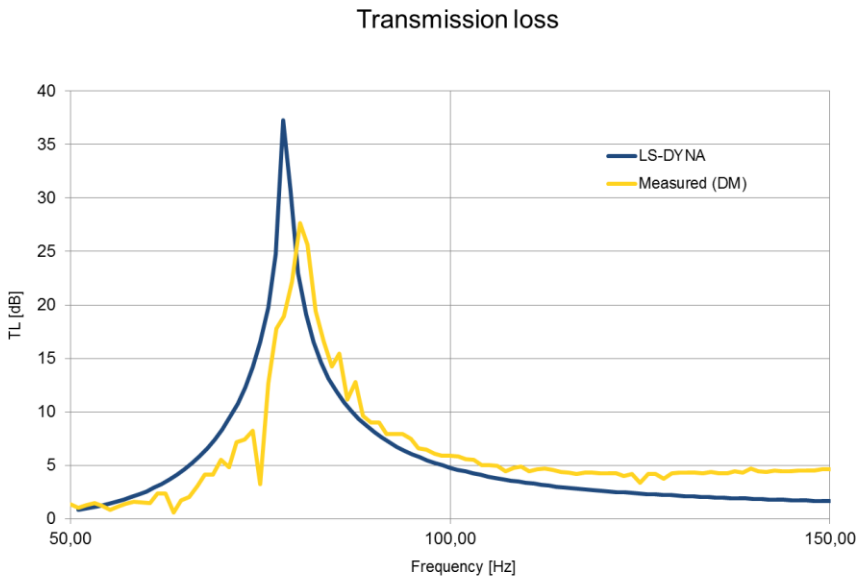

Fig. 8: Detailed comparison between experimental and simulated transmission loss data for the Helmholtz resonator.

5.3 Resonator with discontinuous insertion

Third simulated acoustic component was a simple reactive resonator with discontinuous tube inside (green part in Fig. 9). We simulated and measured two resonators with a 10 mm and 5 mm gap. These types of components need special boundary conditions applied: free edge and multi-connection. Furthermore, collocation BEM method is not suitable for this type of boundary conditions anymore. The BEM method which is used instead is called indirect variational BEM. This method has fully populated system of equations and is consequently more time consuming. Besides, the method does not allow for pressure and impedance boundary conditions to be applied. Instead, only velocity boundary conditions are allowed. The original geometry of the model remained the same. In LS-DYNA only four-pole method of TL calculation from indirect variational BEM results is available at the moment.

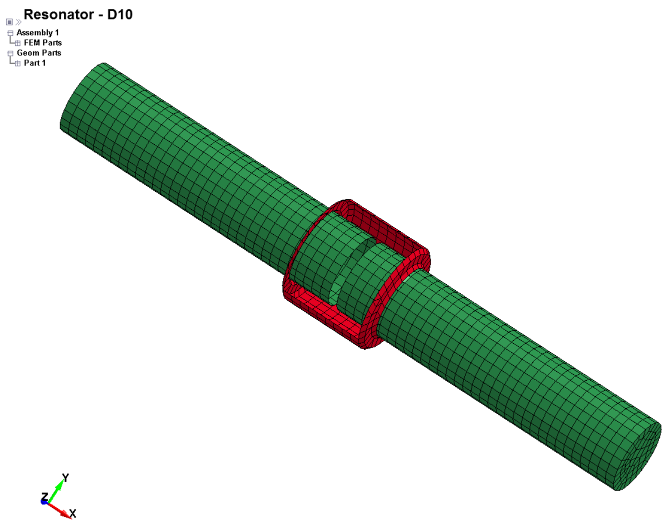

Fig. 9: The resonator with discontinuous insertion assembly.

The simulated TL peak for the 10mm gap case is to be found at 1196 Hz and its TL amplitude is 63 dB. Measured results obtained by the decomposition method for a tested prototype resonator show TL peak of 34 dB at 1106 Hz. The results (Fig. 10) are pretty good since we have to take in consideration that relatively small deviation of CAD geometry from the tested prototype can lead to observable shift in TL peak frequency. Undesired frequency offset between the two peaks is 90 Hz.

Fig. 10: Comparison between experimental and simulated transmission loss data for a disconnected tube resonator with the gap of 10 mm.

The simulated TL peak for the 5 mm gap example is to be found at 1142 Hz with the amplitude of 61 dB (Fig. 11). In this case we have again an approximate match of simulated and measured peaks at 255 Hz frequency offset. Measured results imply the peak TL amplitude of 25 dB at the frequency of 887 Hz.

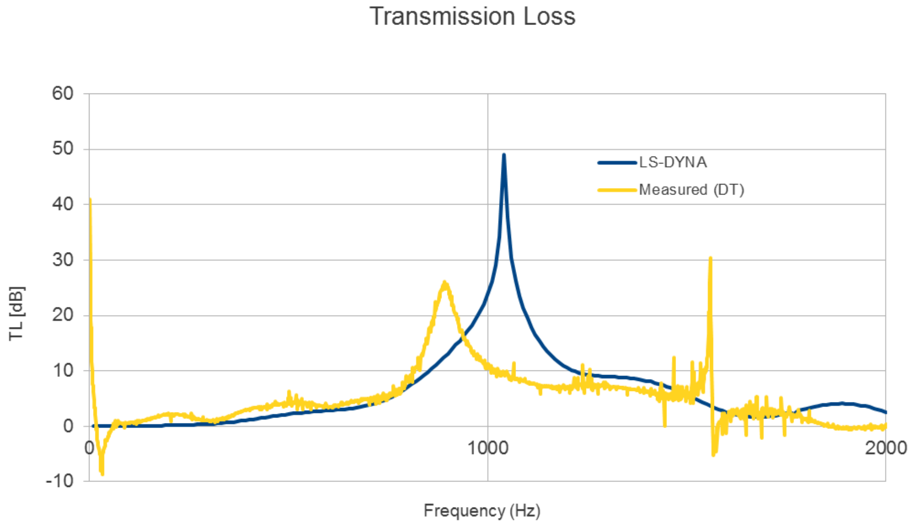

Fig. 11: Comparison between experimental and simulated transmission loss data for a disconnected tube resonator with the gap of 5 mm.

We have to stress in this very place, that a part of the peak discrepancy could come from the differences in dimensions between prototyped and simulated reactive muffler.

6 Summary

The goal of this paper was to present validated simulations of transmission loss in LS-DYNA for basic acoustic elements in exhaust systems. The transmission loss in our case was measured using the decomposition method where the sound pressure may be decomposed in its incident and reflected waves. All acoustic components were modeled and set up in LS-PREPOST. The results of simulations were compared and validated with the measured results.

It was found that LS-DYNA's acoustics BEM capabilities can be of a great help in prototyping process for acoustic elements like expansion chambers, Helmholtz resonators, and certain reactive mufflers. In the case of round expansion chamber and Helmholtz resonator the measured and simulated curves matched perfectly. In the case of reflective muffler discrepancy in the results is present, but we can still consider them as adequate.

7 Acknowledgements

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