Node to node contacts for SPH applied to multiple fluids with large density ratio

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1 Abstract

The interesting and complex behavior of fluids emerges mainly from interaction processes. SPH has shown to be a simple, yet flexible method to cope with many fluid simulation problems in a robust way. However in SPH, particles have a spatial distance (smoothing length) over which their properties are smoothed by a kernel function. Smoothed quantities of a particles show falsified values when densities and masses of neighboring particles vary largely within the smoothing length. The erroneous quantities lead to undesirable effects, reaching from unphysical density and pressure variations to spurious and unnatural interface tensions, and even to severe numerical instabilities. In this paper, instead of using the traditional interaction between SPH parts through SPH interpolation, we present a node to node contact between different SPH parts to avoid the instabilities due to large density ratios at the interfaces. The methods allow the users to select the desired amount of contact force between two SPH parts by choosing the desired penalty scale factors according to the simulation problem at hand. Some examples are tested to show that the method was successfully used to stably simulate multiple fluids with large density contrasts without the above described artifacts apparent in standard SPH simulation.

2 Introduction

Many of the problems of geophysical and industrial fluid dynamics involve complex flows of multiple liquids and gases coupled with heat transfer. The motion of the surfaces of the liquids can involve sloshing, splashing and fragmentation. Thermal and chemical processes present further complications. The simulation of such systems can sometimes present difficulties for finite difference and finite element methods, particularly when coupled with complex free surface motion, while smoothed particle hydrodynamics can easily follow wave breaking, and it provides a reasonable simulation of splash on a length scale exceeding that where surface tension must be included.

SPH is a Lagrangian method for solving partial differential equations. Essentially, the domain is discretized by approximating it by a series of roughly equi-spaced particles. They move and change their properties (such as temperature) in accordance with a set of ordinary differential equations derived from the original governing PDEs. SPH was first applied by Lucy (1977) to astrophysical problems, and was extended by Gingold (1982). Cloutman (1991) used SPH to model hypervelocity impacts. Libersky and Petschk have shown that SPH can be used to model materials with strength. In recent years it has been developed as a method for incompressible isothermal enclosed flows by Monaghan (1994).

When simulating fluids, it is important to capture interaction effects accurately in order to reproduce real world behavior. Smoothed Particle Hydrodynamics has shown to be a simple, yet flexible method to cope with many fluid simulation problems in a robust way. Unfortunately, the results obtained when using SPH to simulate miscible fluids are severely affected, especially if density ratios become large. In SPH, particles have a spatial distance covered by smooth length over which their properties are smoothed by a kernel function. Problems arise when rest densities and masses of neighboring particles vary within the smoothing length, as in such cases the smoothed quantities of a particle show falsified values. The undesirable effects reach from unphysical density and pressure variations to spurious and unnatural interface tensions, as well as severe numerical instabilities.

OTT and SCHNETTER (2003) have derived an adapted continuity equation and compared the sound and shock wave simulation results to analytical solution. Although the results for these specific applications are promising, the use of the standard as well as the adapted continuity equation does not produce stable results for long-term simulations. TARTAKOVSKY and MEAKIN (2005), Hu and ADAMS (2006) used a corrected density summation for their investigations. TARTAKOVSKY and MEAKIN concentrated on miscible flow in fracture apertures with complex geometry and combined a modified SPH flow equation with an advection-diffusion equation. Hu and ADAMS focused on the investigation of numerical examples such as droplet oscillation and deformation in shear flow in 2D and the comparison to analytical solutions. Solenthaler and Pajarola (2008) replaced the density computation in SPH by a measure of particle densities and consequently derived new formulations for pressure and viscous forces. This method enables the user to select the desired amount of interface tension according to the simulation problems at hand. Muller et al (2005) proposed a technique to model fluid-fluid interaction based on the Smoothed Particle Hydrodynamics method. For the simulation of air-water interaction, air particles were generated on the fly only where needed, they also modeled dynamics phase changes and interface forces, those techniques make possible the simulation of the phenomena such as boiling water, trapped air and the dynamics of a lava lamp.

In this paper, instead of using the traditional interaction between SPH parts through SPH interpolation, we present a node to node contact between different SPH parts to avoid the instabilities due to large density ratios at the interfaces, also the SPH interpolation of density and forces was carried out locally inside the physical domain of the each SPH part. The methods allow the users to select the desired amount of contact force between two SPH parts by choosing the desired penalty scale factors according to the simulation problem at hand. Some examples are tested to show that the method was successfully used to stably simulate multiple fluids with large density contrasts without the above described artifacts apparent in standard SPH simulation.

3 Standard SPH formulation

3.1 Fundamentals of the SPH method

Particles methods are based on quadrature formulas on moving particles $(x_i(t), w_i(t)) i \in P$, P is the set of the particles. $x_i(t)$ is the location of particle i and $w_i(t)$ is the weight of the particle i. The quadrature formulation for a function can be written as:

$$\int_{\Omega} f(x)dx = \sum_{j \in P} w_j(t) f(x_j(t)) \tag{1}$$

The quadrature formulation (1) together with the definition of smoothing kernel leads to the definition of the particle approximation of a function. The interpolated value of a function: $u(X)$ at position X using the SPH method is:

$$\prod^h (u(x_i)) = \sum_{j \in \Omega} w_j(t) u(x_j) W(x_i - x_j, h) \tag{2}$$

Where the sum is over all particles inside Ω and within a radius $2h$, W is a spline based interpolation kernel of radius $2h$. It mimics the shape of a delta function but without the infinite tails. It is a C^2 function. The kernel function is defined as following:

$$W(x_i - x_j, h) = \frac{1}{h} \theta \left\{ \frac{x_i - x_j}{h(x, y)} \right\} \tag{3}$$

$W(x_i - x_j, h) \to \delta$ when $h \to 0$, δ is Dirac function, h is a function of x_i and x_j and is the so-called smoothing length of the kernel.

And the cubic B-spline function is defined:

$$\theta(d) = C \times \begin{cases} 1 - \frac{3}{2}d^2 + \frac{3}{4}d^3 & \text{when } 0 \le d \le 1 \\ \frac{1}{4}(2-d)^3 & \text{when } 1 \le d \le 2 \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

The gradient of the function $u(X)$ is given by applying the operator of derivation on the smoothing length:

$$\nabla \prod^h (u(x_i) = \sum_j w_j u(x_j) \nabla W(x_i - x_j, h) \tag{5}$$

Evaluating an interpolated product of two functions is given by the product of their interpolated values.

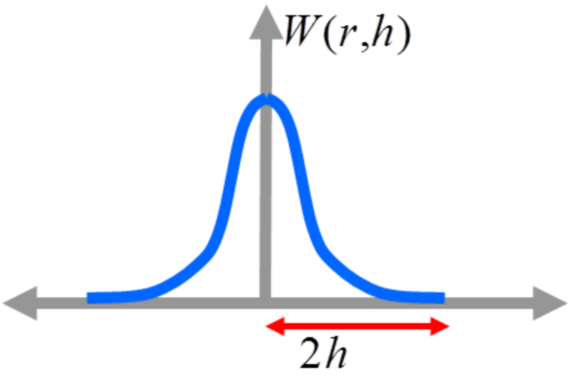


Fig 1. Support size of 2d kennel function

3.2 Continuity equation and Momentum equation

The particle approximation of continuity equation is defined as:

$$\frac{d\rho_i}{dt} = \rho_i \sum_j \frac{m_j}{\rho_j} \left(v^\beta{}_i - v^\beta{}_j \right) W_{ij'\beta} \tag{6}$$

It is Galilean invariant due to that the positions and velocities appear only as differences, and has good numerical conservation properties. $v^\beta{}_i$ is the velocity component at particle i.

The discretized form of the SPH momentum equation is developed as:

$$\frac{dv_i^\alpha}{dt} = -\sum_j \frac{m_j}{\rho_i \rho_j} (\sigma_i^{\alpha\beta} \pm \sigma_j^{\alpha\beta}) W_{ij,\beta} \tag{7}$$

The above formulation ensures that stress is automatically continuous across material interfaces. Different types of SPH momentum equations can be achieved through applying the identity equations into the normal SPH momentum equation. Symmetric formulation of SPH momentum equation can reduce the errors arising from particle inconsistency problem.

From equation (7), the following particle body forces were derived:

$$F_i^{pressure} = -\sum_j m_j \frac{p_i + p_j}{2\rho_j} \nabla W(r_{ij}, h)$$

$$F_i^{viscosity} = \mu \sum_j m_j \frac{v_i - v_j}{2\rho_j} \nabla^2 W(r_{ij}, h) \qquad (8)$$

Where $r_{ij} = x_i - x_j$, μ is the viscosity coefficient of the fluid. The pressure p_i are computed via the constitutive equation:

$$p_i = k(\rho_i - \rho_0) \qquad (9)$$

where k is the stiffness of the fluid and ρ_0 is its initial density.
Finally, for the acceleration of a particle i, we have:

$$\mathbf{a}_i = 1/\rho_i (F_i^{pressure} + F_i^{viscosity} + F_i^{external}) \qquad (10)$$

Where $F_i^{external}$ are external forces such as body forces or forces due to contacts.

4 Multiple Fluids

The above equations (1)--(10) were designed to handle single phase fluid and can be easily extended in order to handle multiple fluids with different rest density. Cares must be taken to avoid the interface instability due to the large density ratio across the fluids interfaces.

4.1 Interaction through standard SPH interpolation

As shown in Fig 2, the standard way to handle the interactions between different SPH parts is through the SPH interpolation functions (i.e treated as one part for multiple SPH fluids) and no contact treatments are needed on the interfaces of the diffferent SPH parts. In SPH, particles have a spatial distance (smoothing length) over which their properties are smoothed by a kernel function (such as density, pressure). Smoothed quantities of a particles show falsified values when densities and masses of neighboring particles vary largely within the smoothing length. As shown in Muller et al (2005), miscible fluids with a density ratio larger than 10 can not be realistically simulated if the standard SPH density summation is used. The reason is that in SPH, the macroscopic flow is mainly governed by the density computation. Over or underestimating the density leads to erroneous pressure values, which might result in unnatural acceleration caused by erroneously introduce pressure ratio (Ihmsen et al 2011). Also lead to a spurious interface tension and a large gap between the fluids. The erroneous quantities lead to undesirable effects, reaching from unphysical density and pressure variations to spurious and unnatural interface tensions, and even to severe numerical instabilities

Another issue with the interaction through standard SPH interpolation is that different SPH fluid parts may stick together after the interaction due to the SPH function interpolations.

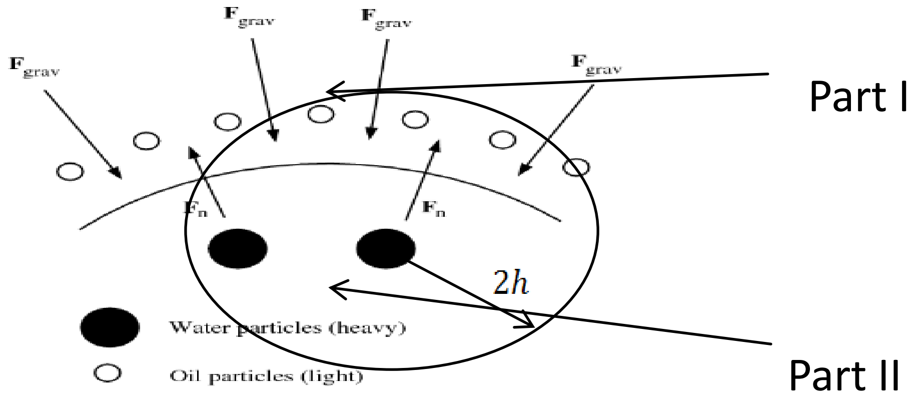


Fig 2. Interaction through SPH interpolation (treated as one part and no contact is needed)

The following example of tank sloshing with water and air shows that when two fluids with different rest densities are mixed, a density gradient and, thus, a pressure gradient will emerge at their common interface. This pressure gradient will cause the less dense fluid to rise inside the denser fluid.

Standard SPH interpolation for high density ratio across the interface

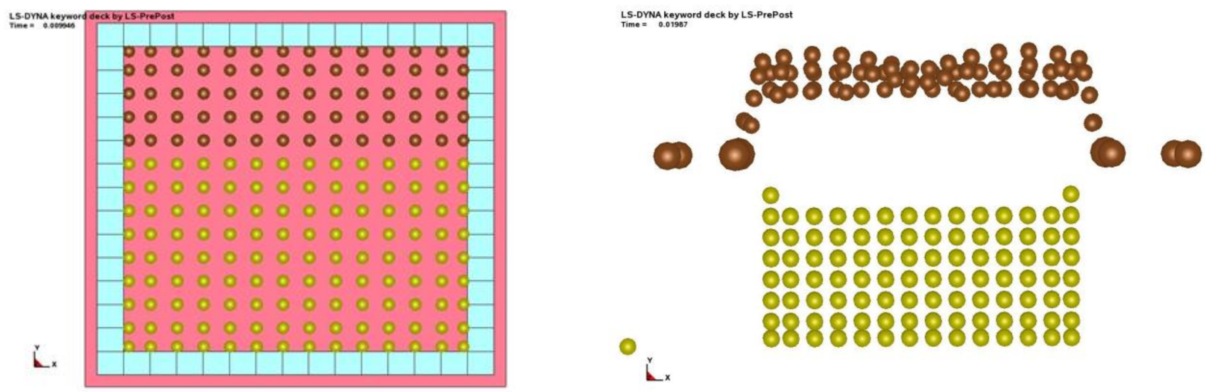


Fig 3.Instability on the interface for tank sloshing with SPH interpolation method

4.2 Interaction through node to node contacts

A penalty based node to node contact model is introduced on the interfaces of the different SPH parts. As shown in Fig 4., all the SPH interpolations (density, pressure and so on) are carried out inside the local domains of each SPH part. No spurious interface tension or interfaces instability happened in this model. The contact forces on the interfaces will be applied to the external forces as in equation (10).

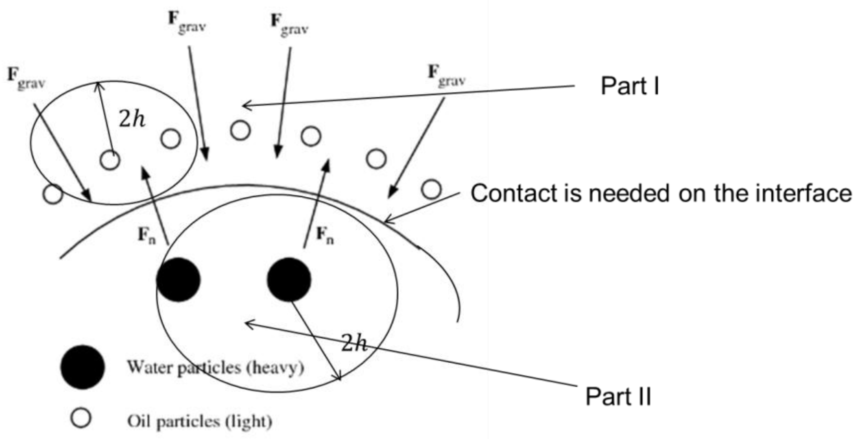


Fig 4. Ineraction through node to node contacts

4.2.1 Penalty contact model for SPH part's interaction

Linear spring dashpot model:

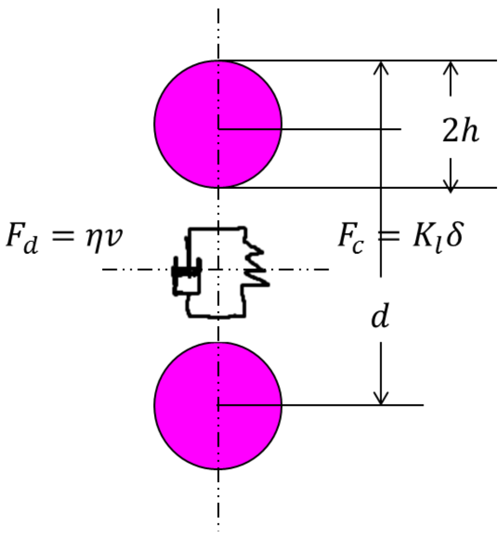


Fig 5. Linear spring dashpot node to node contacts model

Many different contact models have been applied in the particles methods (Vu-Quoc and Zhang 1999). As well as the widely used spring-dashpot models. More complex models have also been proposed. In our practice, we will focused on the two widely used spring-dashpot models.

(Cundall and Strack 1979) model the normal contact through a linear-spring-dashpot system similar to the one shown in Fig 5. for a collision between a particles a flat base.

In this system, the repulsive contact force acting on particle due to contact F_c , is directly proportional to the displacement or overlap between particles δ:

$$F_c = K_l \delta \tag{11}$$

where $\delta = \mathrm{d} - 2\mathrm{h}$ and K_l is the linear-spring constant or stiffness. If the contact is modeled using only this linear-spring, no energy will be consumed and the contact will be ferfectly elastic. In reality, some kinetic energy is dissipated in plastic deformation, and/or converted to heat or sound energy. To account for those energy losses, a contact damping force based on a dashpot model is also included:

$$F_d = \eta v \tag{12}$$

The contact damping force is proportional to the relative velocity of the contacting particles, where the constant of proportionality η is known as the damping coefficient, $v = v_1 - v_2$.

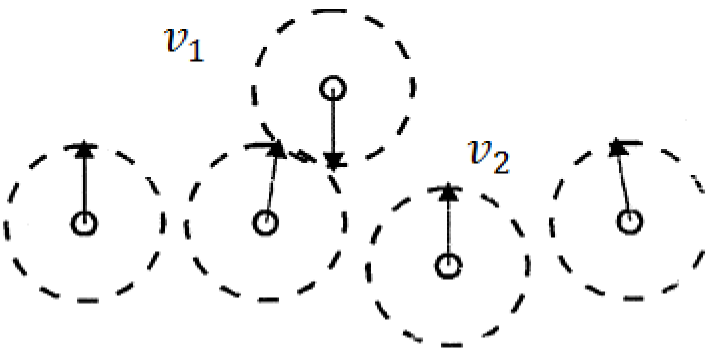


Fig 6. Relative velocity for Linear spring dashpot node to node contacts model

Non-linear Hertzian-spring-dashpot model:

Another commonly used contact model is: non-linear Hertzian-spring-dashpot model.
It differs from the LSD model in that the repulsive contact force F_c is taken to be proportional to the inter-particle overlap to the power 3/2:

$$F_c = K_h \delta^{3/2}, \; F_d = \eta v \delta^{1/4} \tag{13}$$

Where K_h is the non-linear Hertzian-spring constant or stiffness.
Add stiffness contact force F_c and damping contact force F_d into acceleration equation (10) we got:

$$\mathbf{a}_i = 1/\rho_i (F_i^{pressure} + F_i^{viscosity} + F_i^{external} + F_i^{c} + F_i^{d}) \tag{14}$$

Based on spring and mass system, the critical time step for LSD contact model is defined as:

$$\Delta t_c = 2\pi \sqrt{m / K_l} \tag{15}$$

Accordingly, the recommended time step from this work was less than:

$$0.2\pi\sqrt{m/K_l} \tag{16}$$

5 Examples

Water, air impacting with rigid ring

3D tank with fluids which has the dimension of 1.0X0.8X0.01 (Fig. 7) was calculated to validate the node to node contact in LS-DYNA for multiple SPH parts with high density ratio across the interfaces. The fluids in the tank were water and air with air on the top, the density ratio between those two fluids is more than 1000. Both water and air were model with SPH particles. A rigid ring modeled with cylinder shell impacted the fluids in the tank with the speed of 50 in Y direction. The results from the SPH particles were compared with the results from the ALE method with the same dimension and parameters (see Fig.9 and Fig.10).

In the model, automatic_ node_ to_surface contacts were used for the interaction between air, water particles and rigid shells, a node to node contact was used for the interaction between air particles and water particles. The contact between two SPH particles from different parts was detected when the distance of two particles is less than SRAD*(sum of smooth lengths from two particles)/2.0. SRAD is parameter ranged from 0 to 1.0 and is used to adjust the detecting criteria due to initial penetration.

The standard interaction through SPH interpolation will not work for this case. A proper penalty scale factor has to be used for better performance. As show in Fig. 8, a double value of penalty scale factor will cause more noises around the interface of the two SPH fluids. The final deformed shape of water was comparable with the results from ALE elements (Fig. 9). The velocity historys for the rigid ring from both SPH model and ALE model were plotted and compared in Fig. 10, two results were close.

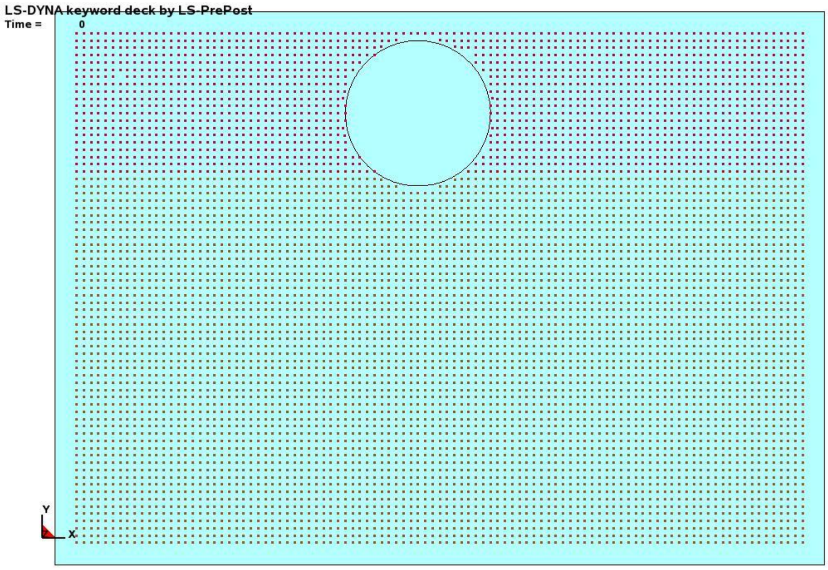
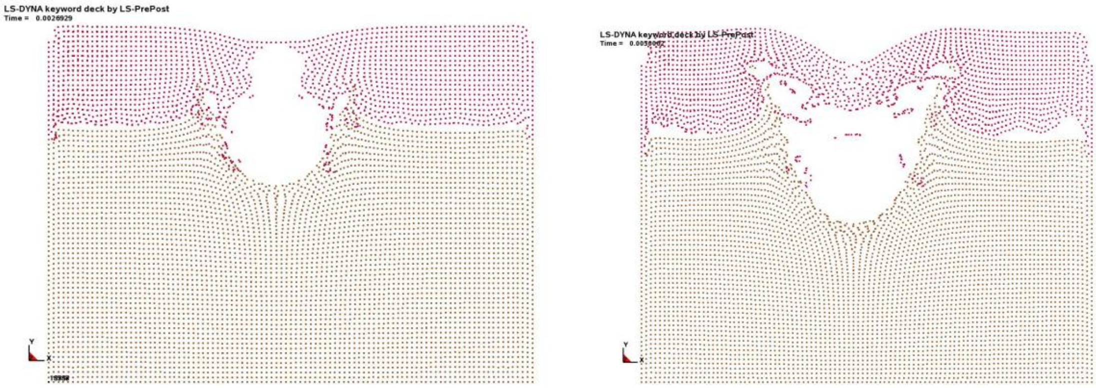
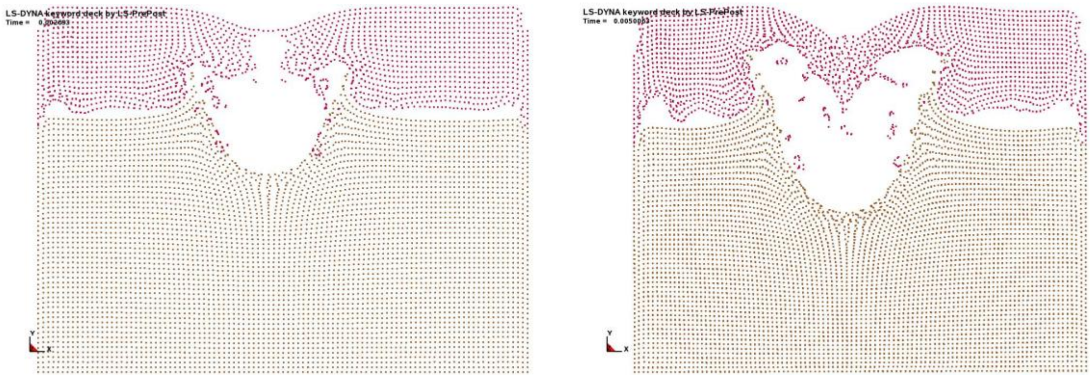


Fig 7. Problem set up of water impact



t=2.7 ms t=5.0 ms



t=2.7 ms t=5.0 ms

Fig 8. Upper: deformation shape for air and water model
Lower: deformation shape with double value of penalty scale factor

Fig 9. Final deformation shape from SPH model (left) compared to ALE model (right)

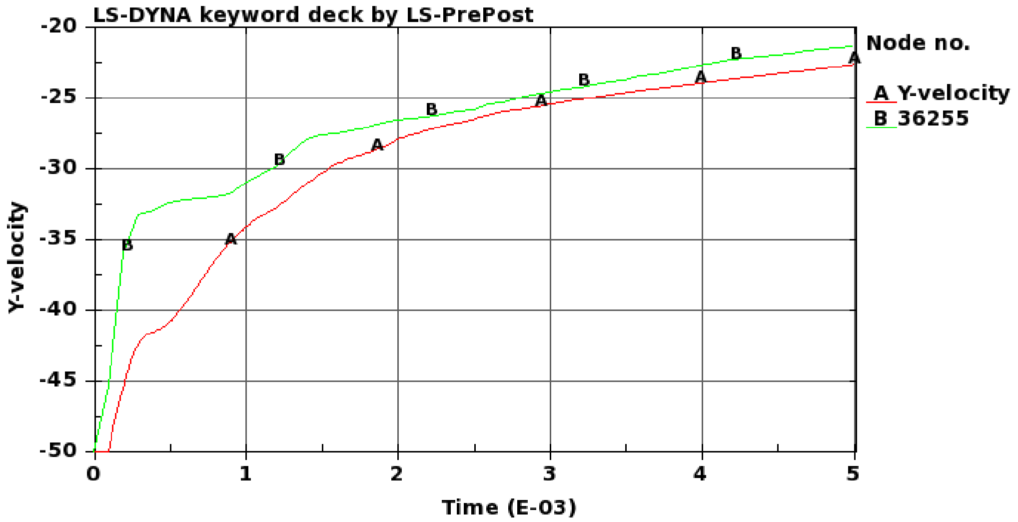


Fig 10. Impact velocity from SPH model (B) compared to velocity from ALE model (A)

6 Summary

We present a node to node contact algorithm for the interaction between different SPH parts to avoid the instabilities due to large density ratios across the interfaces when using the traditional interaction between SPH parts through SPH interpolation. The methods allow the users to select the desired amount of contact force between two SPH parts by choosing the desired penalty scale factors according to the simulation problem at hand. Some examples are tested to show that the method was successfully used to stably simulate multiple fluids with large density contrasts without the above described artifacts apparent in standard SPH simulation. In the future, friction (viscosity) model for node to node contact may be added for the node to node contact option with friction, also some other non-linear contact models may be added.

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