Evaluation of Viscoelastic Material Models in LS-DYNA based on Stress Relaxation Data

Gokula Krishnan M[1], Vesna Savic[2], Bryan Cordeiro[2], Surjayan Biswas[1]

[1]Tata Consultancy Services

[2]General Motors LLC

Abstract

Viscoelastic behaviour of a material is often used as a probe in the field of material science since it is sensitive to the material's chemistry and microstructure. The behaviour enables understanding of the quantity of energy absorbed by the material's internal structure and the energy dissipated to the surroundings. The viscoelastic properties can be determined experimentally by tests such as stress relaxation, creep, or Dynamic Mechanical Analysis (DMA).

Numerical modelling of rubber-like viscoelastic materials in terms of energy dissipation and energy storage is usually done using hyperelastic and viscoelastic constitutive models. Hyperelastic material model captures the material's nonlinear elastic behaviour with no time dependence. Viscoelastic model describes the material response as a function of time, frequency, temperature, and contains an elastic and viscous part.

This paper presents the dynamic characterization of rubber in terms of hyperelastic and viscoelastic constitutive models. The parameters of the constitutive models are determined from the uniaxial tensile and stress relaxation tests. These parameters are used for the numerical model of the rubber components and the accuracy of the characterization is presented by means of a numerical case study.

Capabilities of different constitutive models available in LS-DYNA to predict viscoelastic behaviour of rubbers viz., MAT76 (general viscoelastic), MAT77_H (hyperelastic rubber) and MAT77_O (Ogden rubber) are compared. Additionally, the recent developments under MAT_ADD_INELASTICITY are discussed and are compared with the general viscoelastic model.

Keywords

Elastomers, MAT76, MAT77_O, Hyperelasticity, Stress Relaxation, Viscoelastic behavior, Prony series

Introduction

Elastomers are widely used in many automotive components such as seals, mounts, suspension, gaskets, grommets, bushing, hoses, etc., primarily for their hyperelastic properties. In contrast to metals and some thermoplastic materials, elastomers exhibit nonlinear stress-strain behavior under application of load. They can undergo large deformation under load application and return to their original shape upon removal of the load. This behavior is due to the long coiled polymer chains with a high level of cross linking. Upon application of load, the chains stretch greatly and regain their original shape upon load removal [1].

Elastomers, in addition to their hyperelastic nature, also exhibit viscoelastic behavior. The viscoelastic response of an elastomer is often used as a probe in the field of material science since it is sensitive to the material's chemistry and microstructure. Elastomers such as natural rubber and Ethylene Propylene Diene Monomer (EPDM) for example, exhibit viscoelastic and hyperelastic behavior. Under application of constant strain (deformation) the stress response(reaction force) gradually reduces as a function of time, as evident in a stress relaxation experiment. Conversely, upon application of constant stress (force) the strain (deformation) gradually increases as a function of time, as evident in a creep experiment. It is imperative that the material characterization and the numerical modeling procedure encompass the hyperelastic and viscoelastic properties of elastomers. Failure to do so may result in an underprediction of the material's compliance.

Modeling of the linear and non-linear behavior of elastomers is essential for accurate representation of these materials in Finite Element (FE) simulation. Attempts by Bergström and Boyce show that mechanical behavior can be decomposed into two parts. The first part is an equilibrium network corresponding to the state approached in long-time stress relaxation tests, and the second part is the time dependent deviation from the equilibrium [1]. Reese and Govindjee proposed a model based on a multiplicative decomposition of the deformation gradient into an elastic and an inelastic part [2], a continuation of work proposed by Lubliner [3]. Currently, there are different approaches to characterize the viscoelastic behavior of elastomers. Arruda and Boyce [4] used an 8-chain constitutive model to successfully capture the uniaxial tension, biaxial extension, uniaxial compression, plane strain compression and pure shear. The model accurately captures the cooperative nature of network deformation while requiring only two material parameters, an initial modulus and a limiting chain extensibility. A mathematical model has also been proposed for relaxation modulus and its numerical solution. The model formula is extended from sigmoidal function considering nonlinear strain hardening [5]. There are other models that have presented the dashpot's stress as a nonlinear function dependent on the strain rate [6]. Qinwu Xu et al. [7] have used an elastic network–viscous medium system with five model parameters to represent general materials considering nonlinear strain hardening. Subsequently they developed a robust numerical algorithm to implement the model for simulating dynamic responses. Koontz et al. [8] had fitted stress relaxation data in Matlab with a Prony series using a unique parameter input method developed specifically for this analysis. In continuation with the cited studies, a more methodical and practical approach to modelling viscoelastic properties of elastomers in LS-DYNA and the capabilities of different constitutive models are discussed.

Fundamentals of Viscoelasticity

Viscoelasticity is a fundamental property exhibited by materials that display elastic and viscous deformation when subjected to various loading conditions. Unlike linear elastic materials, the stress-strain response for viscoelastic material is history-dependent, that is the stress is not just a function of the current strain, but also of the rate of change in strain or strain rate.

The presence of viscoelasticity in a material is obvious in a stress relaxation experiment as shown in Figure 1. An elastomer specimen is stretched to a certain strain instantaneously (ε_o), the internal stress (σ_o) generated due to the strain is recorded as a function of time. The entangled cross linked polymer chains in the elastomer resist the strain with high stress initially [9]. As time progresses, the chains have sufficient time to untangle themselves and stretch leading to reduction in stress. After a certain time, the reduction in stress becomes insignificant leading to a stress state called equilibrium stress(σ_∞). The instantaneous and equilibrium relaxation moduli are calculated as shown in Equations 1 and 2, respectively.

$$E(t = 0) = \frac{\sigma_{t=0}}{\varepsilon_0} \tag{1}$$

$$E(t = \infty) = \frac{\sigma_{t=\infty}}{\varepsilon_0} \tag{2}$$

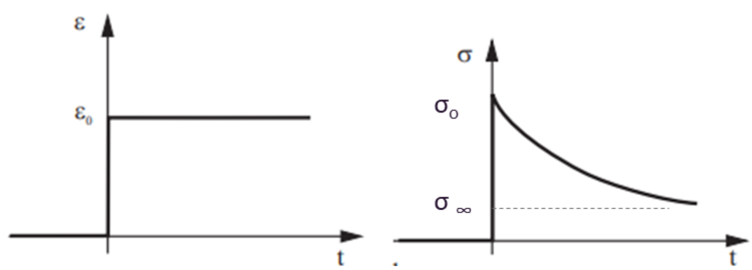


Figure 1: Stress and strain response for a stress relaxation experiment

Stress relaxation experiments were used in this study to calibrate the viscoelastic material models and will be discussed in more detail in subsequent sections of the paper.

Numerical Modelling of Viscoelastic Materials

Hyperelastic material modelling

A hyperelastic material is defined by its elastic strain energy density, W which is a function of the elastic strain state. The formulation provides a non-linear relation between the stress and strain as shown in Equation 3.

$$W = \sum_{i,j=1}^{3} \int_0^{\epsilon ij} \sigma_{ij} \, d\epsilon_{ij} \tag{3}$$

The strain energy density function can also be expressed as a function of principal stretches (λ_i) or invariants (I_i) of left Cauchy-Green strain tensor (B), as shown in Equation 4. Stress-strain relations are derived by differentiating the strain energy density function.

$$W = W(F) = W(\lambda_1, \lambda_2, \lambda_3) = W(I_1, I_2, I_3) \tag{4}$$

Several hyperelastic material models have been developed to predict the behavior of elastomers. Popular material models used in commercial finite element(FE) solvers are Mooney-Rivlin, Neo-Hookean, Ogden, Yeoh, and Arruda-Boyce model.

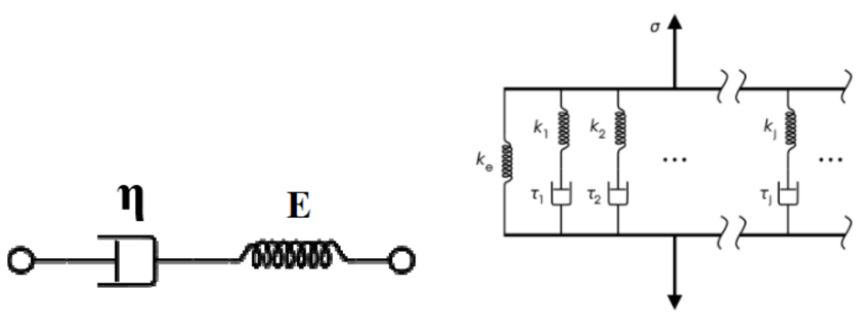
The present work utilizes Ogden hyperelastic material model to define the behavior of the tested natural rubber. The Ogden hyperelastic model [10] derives the strain energy density function, W in terms of generalized strain. The model is based on the three principal stretches (λ_1, λ_2, λ_3) and 2N material constants, where N is the order of polynomial that constitutes the strain energy density function, defined as

$$W = \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i}\left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3\right) + \sum_{k=1}^{N} \frac{1}{D}(J-1)^{2k} \tag{5}$$

In Equation 5, μ_i and α_i are the material constants, such that the stability condition $\mu_i\ \alpha_i > 0$ for all i=1,N is satisfied. J is the determinant of the strain gradient tensor, and D is a material constant related to the bulk modulus.

<u>Viscoelastic modelling</u>

The viscoelastic material is modeled as the combination of viscous and elastic elements. The elastic element is denoted by a spring (E) which follows Hooke's law, and it returns to its original shape (zero strain) upon removal of stress. The response is instantaneous within the elastic element. The viscous element is denoted by a dashpot (η). The viscous element stress is proportional to the strain rate. A popular viscoelastic model to represent elastomers is the Maxwell model where a spring and a dashpot are connected in series, as shown in Figure 2a.



(a) (b)

Figure 2. Representation of a) Maxwell model, b) generalized Maxwell model.

Upon application of strain, the spring element deforms instantaneously providing the initial stress reported in a stress relaxation test. The dashpot needs additional time to deform. As time progresses, the dashpot deforms and compensates for the deformation in spring, thus leading to a zero stress. However, the stress in a stress relaxation test for an elastomer never reaches zero.

Overcoming the limitations of the Maxwell model, which predicts a zero stress at an infinite time, a generalized Maxwell model takes into account that relaxation does not occur at a single time but at a distribution of times which may be due to the presence of polymer chains of varying lengths. The smaller chains contribute to quicker relaxation while longer chains contribute to longer relaxation times. The generalized Maxwell model illustrates this by having as many spring-dashpot elements as are necessary to accurately represent the distribution, as shown in Figure 2b. The relaxation function of a generalized Maxwell model is typically modeled using Prony series when the number of elements, N ≥ 1.

Prony Series

The viscoelastic behavior of a material can be represented by combining a hyperelastic model with a suitable mathematical model to add the effect of time. The mathematical model implemented in LS-DYNA is shown in Equation 6,

$$\sigma_{ij} = \int_0^t G_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau}\partial\tau \tag{6}$$

where G_{ijkl} is the relaxation function for the different stress measures. An important point to consider here is that this viscoelastic stress is added to the stress determined by the chosen hyperelastic material model.

The relaxation function G(t) can be represented as a Prony series, as shown in Equation 7.

$$G(t) = \sum_{m=1}^{N} G_m e^{-\beta_m t} \tag{7}$$

In Equation 7, $G_m(t)$ are the time dependent shear moduli. The unknown coefficients, G_m and β_m can be obtained by choosing the number of Prony series terms and fitting Equation 7 to the data from a relaxation test. Conversely, the curve fitting algorithm available in LS-DYNA can also be used to fit the Prony series coefficients, as discussed in later sections. Theoretically, the experimental data will be more accurately fitted with a higher number of Prony series terms. However, the large number of terms adds computational cost and for most applications 4-6 Prony series terms provide high accuracy of the model.

Experiment Details

Material Details

For this study, a natural rubber with a Shore A hardness of 50 and a density of 1200kg/m^3 was chosen. Natural rubbers are extensively used in automotive industry and have a wide range of applications due to their elasticity, tensile strength, abrasion and chemical resistance. They are commonly used in tires, seals, gaskets, and suspension components. Natural rubbers are also biodegradable and are a renewable resource making them environmentally friendly compared to synthetic alternatives.

Stress Relaxation Testing

Natural rubbers are typically viscoelastic and exhibit a stress relaxation behavior when exposed to a constant load over a period of time. Stress relaxation tests were performed at multiple strain levels using ASTM D412 Die D [11] tensile specimens. The specimens are pulled in tension at a constant speed until the desired strain has been reached. Subsequently, the specimens are held at the desired strain for 2000s. The force (stress) is recorded as a function of time. Tested strain levels include 0.05, 0.1, 0.25, 0.5 and 0.75 mm/mm.

Test Results

Stress relaxation test results for all tested strain levels are shown in Figure 3. The normalized test data shows high instantaneous stress at time t=0. The stress reduces as the time increases and reaches an equilibrium before the end of test.

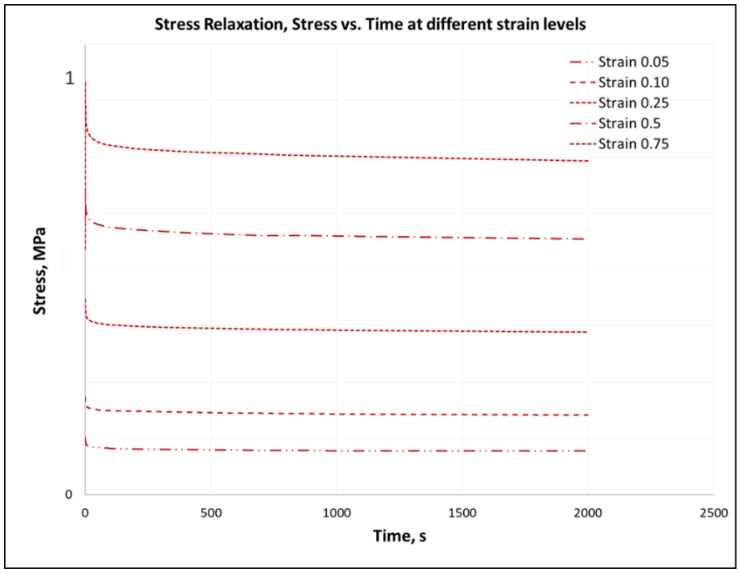


Figure 3. Stress vs. time for strain levels (mm/mm) 0.05, 0.10, 0.25, 0.50, 0.75

Material Modeling

The material models in LS-DYNA, viz. MAT76 (general viscoelastic), MAT77_H (hyperelastic rubber) and MAT77_O (Ogden rubber) are compared in this study.

MAT77_O (Ogden Rubber)

The parameters considered for hyperelastic-viscoelastic simulation using MAT77_O material model are listed in Table 1.

Card 1	
MID	Material identification ID
RO	Mass density
PR	Poisson's Ratio
N	Order of fit for hyperelastic part (Ogden) for LCID1
NV	Number of Prony series terms for viscoelastic part (Maxwell) for LCID2
Card 3a	
SGL/SW/ST	Specimen gauge length/Specimen width/Specimen thickness
LCID1	Load curve ID (engineering stress as function of engineering strain if SGL/SW/ST is set to 1)
DATA	Type of experimental data
LCID2	Load curve ID of the stress relaxation curve

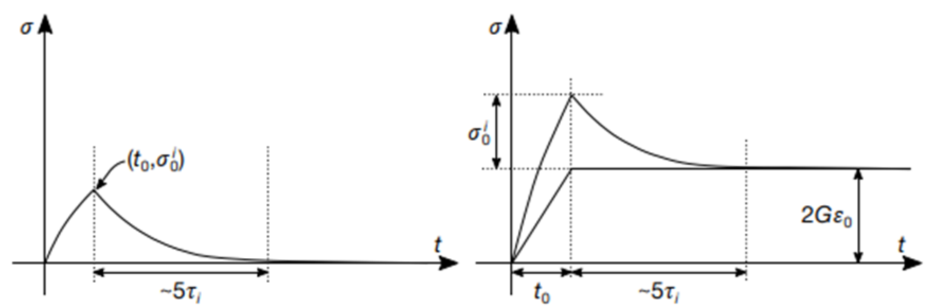
Table 1. Parameters for Ogden rubber model (MAT77_O) [12]

Among the applicable parameters, mass density, Poisson's ratio, and LCID1 can be derived from the test data. Hyperelastic Ogden order(N) can be set from 1-8, however order of 2-3 provides best fit in most cases. Number of Prony series terms (NV) can be set depending on the required fit with relaxation test data. SGL/SW/ST can be set to unity if engineering stress and engineering strain data is entered in the LCID1 field. Type of experimental data must be entered in DATA field (such as 1 - Uniaxial tension, 2 - Biaxial tension, 3 - Pure shear).

A critical aspect should be considered while entering the data for LCID1. In LS-DYNA, the viscoelastic stress is added to the hyperelastic stress as shown in Equation 8.

$$\sigma_{total} = \sigma_{hyperelastic} + \sigma_{viscoelastic} \tag{8}$$

Figure 4, reproduced from the LS-DYNA manual [12], illustrates this implementation. Purely viscoelastic part decays over time to zero strain, as shown in Figure 4a. When hyperelastic and viscoelastic components are added together, as shown in Figure 4b, the resulting behavior corresponds to the behavior of the tested material (Figure 3) for which the equilibrium stress is not zero. Additive nature of the hyperelastic-viscoelastic model implementation requires that the engineering stress vs. engineering strain curve representing the hyperelastic part of the model (LCID1) must correspond to the equilibrium stress (stress at 2000s – refer Figure 3 at x=2000s).



a) Viscoelastic material b) Viscoelastic + hyperelastic material

Figure 4. Contribution of a) viscoelastic and b) viscoelastic + hyperelastic components in MAT077_O

In addition, the viscoelastic implementation with Prony series requires that the stress values recorded during the stress relaxation experiments are scaled before they can be used in the material model. The ordinate of the input curve LCID2 thus represents the effective stress/3x effective strain from the tests. For example, to provide stress relaxation as a function of time for a strain level of 10%, the effective strain is 0.1; considering the elastomer to be nearly incompressible, the ordinate has to be scaled by 1/(3*.01) i.e., 3.33.

Upon fitting all model parameters from the test data shown in Figure 3, the material model is validated by comparing the test results with the results of the simulation of the stress relaxation test. The finite element model of the ASTM D412 Die D dog bone specimen used for the model validation is shown in Figure 5. A hexahedral mesh was used to represent the specimen. Shell elements were not considered due to the complex geometry of the actual part in the vehicle as the complex shapes are better captured by solid elements. The moving end is displaced until the strain corresponding to the tested strain is attained and is then held at that fixed displacement for the duration of the simulation. The opposite fixed end is fully constrained.

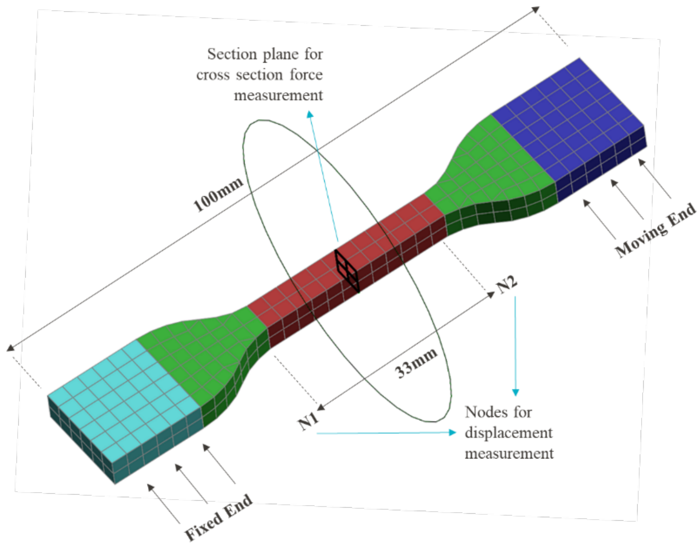


Figure 5. ASTM D412 Die D [11] tensile specimen FE model meshed with hexahedral elements.

The section force from the plane in the middle of the gauge area of the specimen (Figure 5) is used to compute the stress from the simulation. The stress response from the simulation is plotted against the test data for two strain levels in Figure 6. It is evident that a good correlation between test and simulation was achieved for 0.05 and 0.10 strain levels. Results for other strain levels have been omitted for brevity.

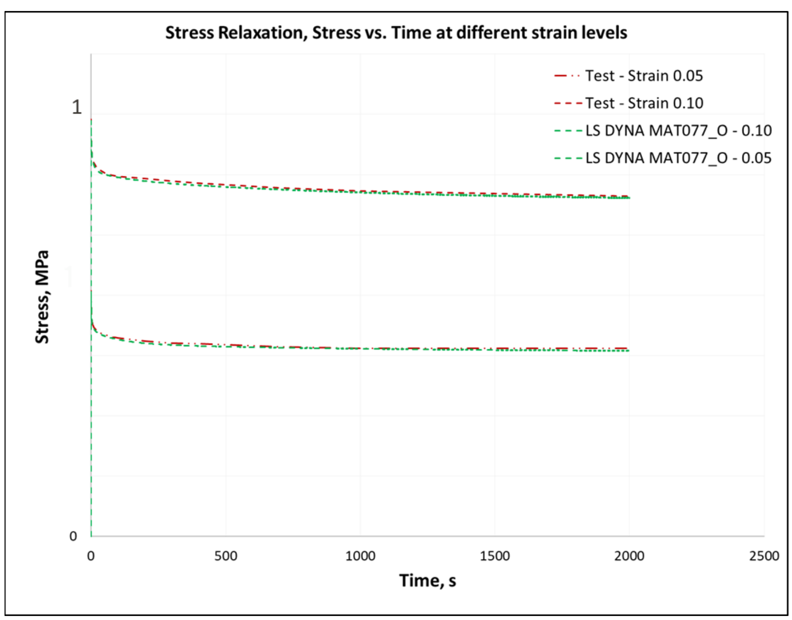


Figure 6. Test-LS DYNA correlation for tested strain levels

MAT76 (General Viscoelastic)

MAT76 material model provides a general viscoelastic Maxwell model having up to 18 Prony series terms. The methodology to provide input relaxation stress data is the same as for MAT77_O, however the hyperelastic stress calculation based on the strain energy density is not available in this material model. Instead, a linear true stress-true strain relationship is assumed and the viscoelastic stress is added to the linear stress function. The stress relaxation prediction from MAT76 material model is the same as from the MAT77_O model since both material models are based on the Maxwell model. A comparison between stress relaxation response of MAT77_O and MAT76 is shown in Figure 7.

MAT77_H (Hyperelastic Rubber)

The implementation of viscoelasticity in MAT77_H is similar to the implementation in MAT77_O, however the two material models differ in the way they characterize hyperelastic part of the total stress. Up to six parameters can be fit to the hyperelastic potential function of MAT77_H while MATT_O allows for the Ogden potential function of order 8 (16 parameters). It is interesting to note that, if fitted with two coefficients only, the hyperelastic part of MAT77_H reduces to the Mooney-Rivlin model. MAT77_H material model offers an enhanced capability when compared to MAT76 material model, as it can predict nonlinear monotonic stress-strain response. However, when it comes to predicting the viscoelastic response MAT77_O, MAT77_H and MAT76 exhibit similar behavior. Figure 7 provides a comparison of the stress relaxation responses of MAT77_H, MAT77_O, and MAT76 models with the test data.

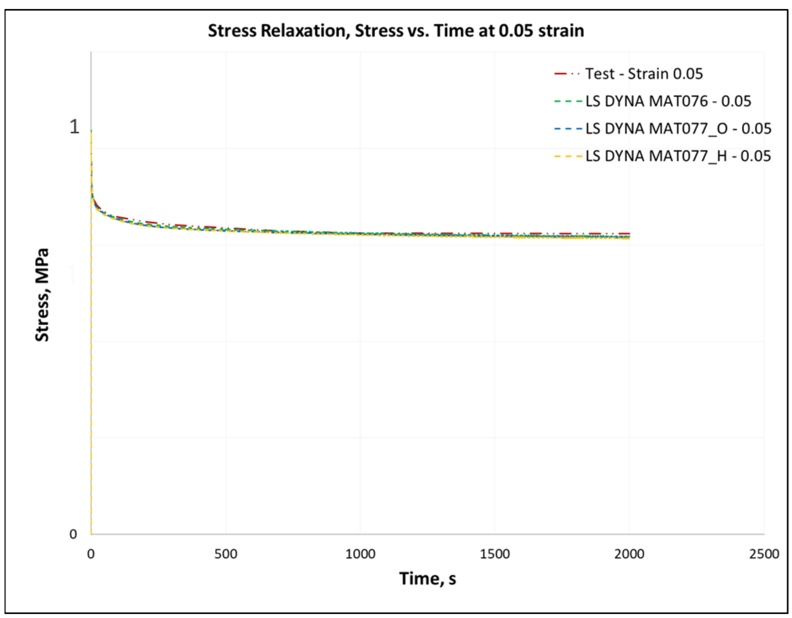


Figure 7. Comparison between MAT77_H, MAT77_O and MAT76 material models

In the absence of the hyperelastic stress component, the viscoelastic stress component exhibits complete decay to zero for all material models under consideration (MAT77_H, MAT77_O, MAT76), as demonstrated in Figure 8 for MAT77_H.

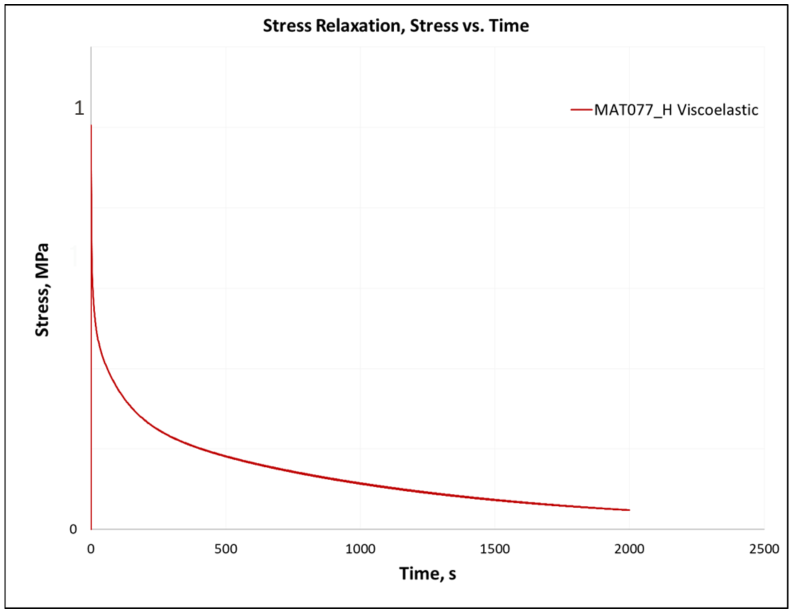


Figure 8. Stress relaxation in the absence of hyperelastic component of total stress

MAT_ADD_INELASTICITY

MAT_ADD_INELASTICITY is an add-on to the existing elastic material models such as MAT_ELASTIC, for example. The model provides options to add isotropic hardening plasticity, creep, and viscoelasticity to an elastic material model. The relevant material parameters to model viscoelasticity using MAT_ADD_INELASCITIY are shown in Table 2.

Card 1	
MID	Material identification ID
NIELINKS	Number of Maxwell links
Card 4	
NIELAWS	Number of elasticity laws (such as viscoelasticity)
WEIGHT	Weight of each Maxwell link
Card 5	
LAW	Isotropic hardening plasticity/creep/viscoelasticity
MODEL	Hypoelasticity or Hyperelasticity in case of elastomers

Table 2. Parameters for MAT_ADD_INELASTICITY [12]

Among the applicable parameters, MID needs to be set to the material ID of the elastic or hyperelastic material model. NIELINKS is the number of Prony series terms used to fit the viscoelastic part of the model. NIELAWS can be set to 1, if viscoelasticity is the only inelasticity to be predicted by the model. Weight for each Maxwell link is entered as shown in Equation 9.

$$w_i = \frac{G_i}{G} \tag{9}$$

G_i is the shear modulus of each Maxwell link, G is the shear modulus of the material model as defined in the elasticity part.

The two material models shown in Figure 9a and Figure 9b are the same. The behavior of MAT_GENERAL_VISCOELASTIC is similar to MAT_ELASTIC with MAT_ADD_INELASTICITY option. Similar approach can be used to model multiple Maxwell links to represent viscoelasticity with any applicable material model.

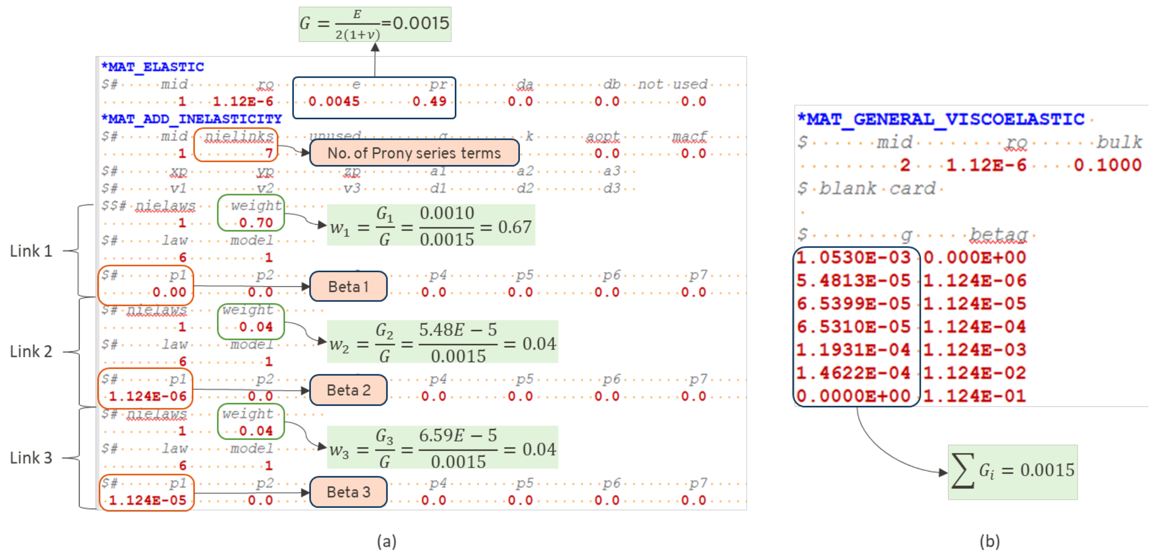


Figure 9. (a) MAT_ELASTIC with MAT_ADD_INELASTICITY, (b) MAT_GENERAL_VISCOELASTIC

Summary/Conclusion

In this work, a natural rubber used for automotive applications has been tested for its viscoelastic properties using a stress relaxation test at multiple strain levels and the results were presented. Three material models available in LS-DYNA viz. MAT76 (general viscoelastic), MAT77_H (hyperelastic rubber) and MAT77_O (Ogden rubber) were compared in this study to characterize the hyperelastic and viscoelastic behavior of elastomers. The viscoelastic material models follow generalized Maxwell approach and have a viscoelastic and a hyperelastic component. In the absence of the hyperelastic stress component, the viscoelastic stress component exhibits complete decay to zero for all material models under consideration. Steps to determine the Prony series terms and its usage with hyperelastic material model MAT77_O were reviewed. General discussion on MAT_ADD_INELASTICITY was also presented, and an example of its similarity with MAT76 (general viscoelastic) was provided.

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