An Engineering Approach to Estimating Partially Saturated Soil Constitutive Properties Using LS-DYNA®

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Abstract

Soil is perhaps the most common civil engineering material, and ironically one of the most difficult to model due to its variability. At any given site, soil samples taken at different depths and distances may show considerable variability. Even if such samples show relative uniformity, there is often the question of what happens if it rains and the soil saturation changes?

In part because of soil's variability, experimental research involving soil often uses dry sand that is widely available and has been characterized sufficiently for use in constitutive models. Any soil constitutive model has two main components:
1. *Compaction response, i.e. pressure versus volume strain response*
2. *Shear strength as a function of confining pressure, i.e. frictional material behavior.*

This manuscript provides an engineering model for both of these constitutive components, such that given a representative constitutive response of dry sand, an estimate of the constitutive properties of a partially or fully saturated version of that dry sand can be made.

Depending on the loading rate and pressure, one or the other of these two constitutive components will be dominant. For high rate loading, e.g. buried charges, pore water does not have time to migrate and remains trapped in partially, or fully, water filled voids in the soil. For this loading rate the compression behavior is dominant and may be defined by a single Equation of State.

Shear strength of sandy soils varies inversely with saturation due to the lubricating effect of water on the interlocking sand particles. At low loading pressures where shear strength dominates, e.g. far-field ground "shock" (stress) propagation and soil-structure interaction, a simple bi-linear pressure-dependent yield surface may adequately capture the sandy soil behavior.

Compaction Model

Figure 1 shows an illustration of two springs in parallel used to represent the compaction behavior of partially saturated soil. This two spring model was motivated by a similar three spring model proposed by Barsotti et al. (2016).

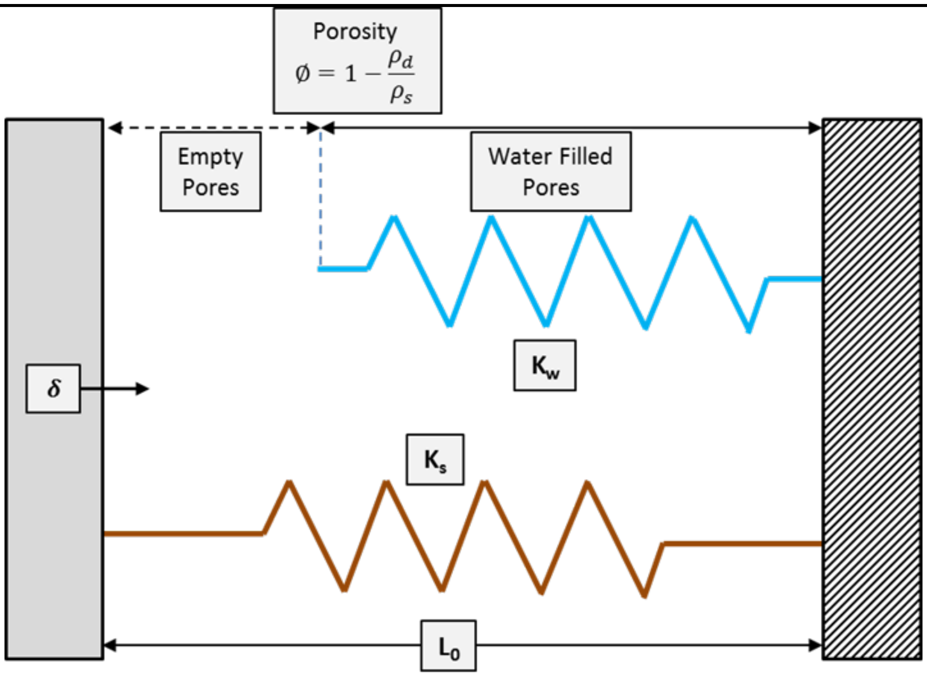

Figure 1 Two spring model of unsaturated soil compaction.

The lower spring with stiffness represents the compaction of the dry soil and the upper spring with stiffness represents the compaction of water. The water spring represents the pores (voids in the soil) that are filled with water. The empty pores must be fully compresses before the water filled pores will be compressed.

In this one dimensional representation of compaction, the applied force is a proxy for the pressure in a hydrostatic compression test and the change in length of the soil spring represents the volume strain, i.e.

$$\varepsilon_{kk} = \ln\left(1 - \Delta L / L_0\right) \quad (1)$$

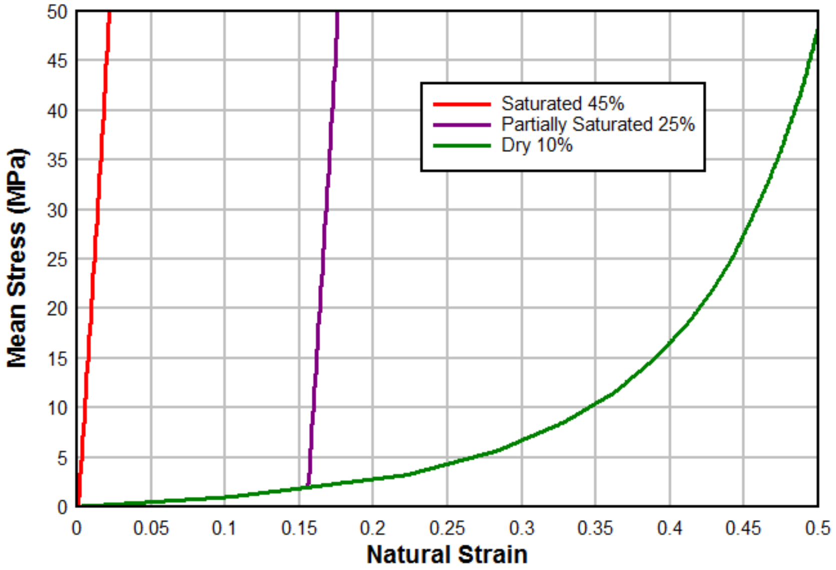

Figure 2 Illustration of saturated, partially saturated and dry soil compaction.

Figure 2 is an illustration of the two extremes for soil compaction of saturated and dry soil with an intermediate case of partially saturated soil. For the saturated case there are no empty pores so both the water and soil springs are engaged simultaneously. For the dry soil case all the pores are empty so there is no water spring and only the soil spring provides resistance to compaction. For the partially saturated case, the empty pores are compacted without resistance, so the dry soil compaction response is followed. Once the water filled pores are engaged both the water and soil springs provide resistance to additional compaction.

The compaction curve for water is obtained using the LS-DYNA *EOS_GRUNEISEN with parameters: C=1480 m/s, $S_1 = 1.92$ and $\gamma_0 = 0.1$ combined with *MAT_NULL that provides the density of water.

The compaction curve for dry soil must be obtained from laboratory hydrostatic compression testing. Unfortunately, dry soil is seldom tested as some moisture is needed to help mold and fill the test specimens. The addition of water to such “dry” test samples is prescribed by the Procter Compaction Test as defined by the American Society for Testing Materials (ASTM D698 & D1557).

> “The Proctor compaction test is a laboratory method of experimentally determining the optimal moisture content at which a given soil type will become most dense and achieve its maximum dry density.”
> Wikipedia

Also see the derivation of the dry density ρ_d in a subsequent section.

Partially Saturated Compaction Algorithm

The basic idea of the algorithm is to combine the two spring stiffnesses accounting for the empty and water filled voids. This is accomplished by shifting the start of the water spring to when the empty void cells have been compacted.

The saturation ratio, S_w, provides the percentage of voids that are filled with water. It follows that the percentage of voids that are empty is $1 - S_w$. The total amount of voids, or porosity, ϕ, is given by

$$\phi = \frac{V_v}{V} = 1 - \frac{\rho_d}{\rho_s} \qquad (2)$$

where V_v is the volume of voids and V is the total volume. Also, ρ_d is the dry density and ρ_s is the density of the soil grains, often taken as 2640 kg/m ^{3}

Thus the water-spring should be engaged when the empty voids are collapsed

$$\frac{\Delta V}{V_0} = \phi\left(1 - S_w\right) \qquad (3)$$

or in terms of the natural volume strain

$$\ln\left(1 - \frac{\Delta V}{V_0}\right) = \ln\left[1 - \phi\left(1 - S_w\right)\right] \qquad (4)$$

In the two spring model LS-DYNA implementation, the water-spring volume strains are offset by the amount of volume strain given by Equation (4), i.e. the *DEFINE_CURVE parameter $\mathrm{OFFA} = \ln\left[1-\phi\left(1-S_w\right)\right]$.

Compaction Model Illustration

Laboratory provided hydrostatic compression data for a sandy soil at three water ratios: 10, 25 and 45%, with the first called “dry” and the last called “saturated.” The model input parameters are:

- Grain (particle) density 2.641×10^{-3} g/mm^3
- Water density 1.000×10^{-3} g/mm^3
- Water pressure versus natural volume strain (from EOS)
- “Dry” soil pressure versus natural volume strain [1]
- Water ratio
- Soil dry (no water) density
- Soil density as tested (“wet”)
- Soil spring length $L_0 = \phi$ (see footnote [2])

Table 1 Three soil saturations hydrostatically tested.

Description	Water Content (%)	Saturation (S_w)	Dry Density (kg/m^3)	Wet Density (kg/m^3)
Dry	10	0.23	1219	1342
Partially Saturated	25	0.57	1219	1525
Saturated	45	1.02	1219	1770

Table 1 provides the reported water percentage and densities of the tested soil. Figure 3 shows the corresponding data with the two spring model comparison for the three water ratios.

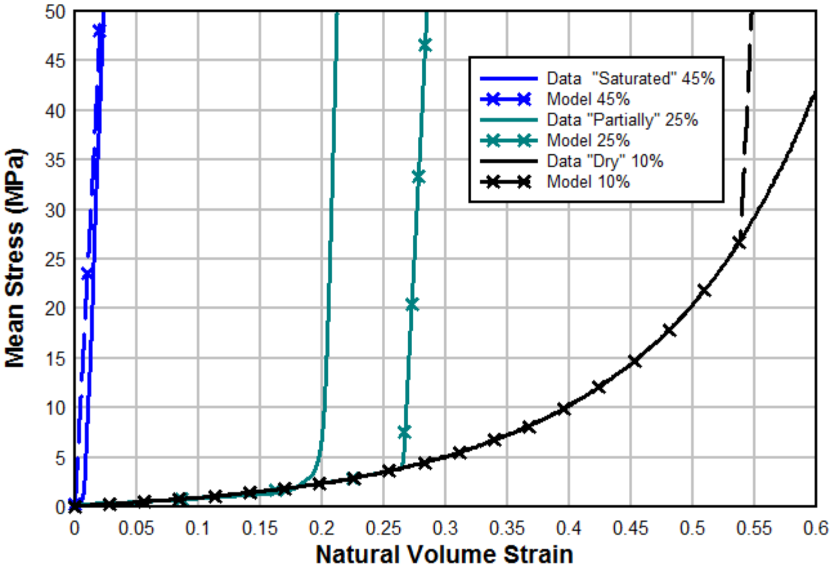

Figure 3 Hydrostatic compression data and two spring model comparisons.

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[1] If “dry” soil pressure versus volume strain is not available from a laboratory test, the data provided by Laine and Sandvik (2001) may be used.

[2] A spring length equal to the porosity is an “over kill” for the saturated case where the offset OFFA is near zero. A more general spring length of OFFA + 0.2 is recommended.

The model reproduces the saturated data as the computed water curve offset, from Equation (4), is only 0.01 because all the pores are water filled. For the partially saturated case with 25% water content the model over predicts the water curve offset as 0.265 rather than the measured 0.2, or an error of about 33%. Finally, for the dry 10% water case the model under predicts the water curve offset as 0.54 compared to the measured 0.6 or an error of -10%.

As this is the only laboratory data used to compare with the model, it is inclusive if the model would perform better or worse for a different soil.

Saturation Modification of Shear Failure Surface

Figure 4 shows the laboratory shear failure data for the saturation levels corresponding to the above compaction data. As indicated by Anderson et al. (2010), the addition of water to dry sand dramatically reduces the shear strength. In the present case, the presence of the water essentially eliminates the frictional response of the sand.

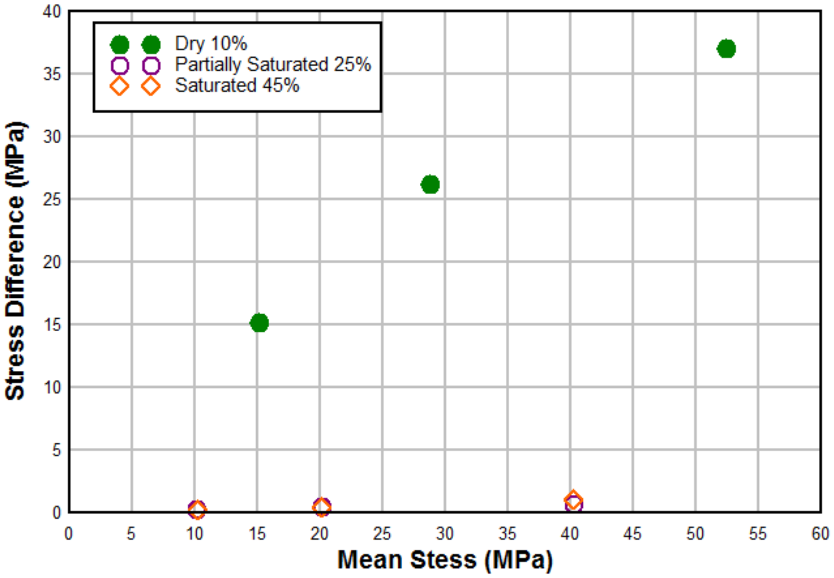

Figure 4 Shear failure data for three degrees of saturation.

References

Barsotti M; E. Sammarco and D. Stevens, (2016), "Comparison of Strategies for Landmine Modeling in LS-DYNA with Sandy Soil Material Model Development," 14th International LS-DYNA Users Conference, Dearborn, MI.

Laine, L. and A. Sandvik, (2001). "Derivation of Mechanical Properties for Sand," CI-Premier PTE LTD, 4th Asia-Pacific Conference on Shock and Impact Loads on Structures, Singapore, pp. 361-368.

Anderson, C.E., T Behner, C.E. Weiss, S. Chocron, R. P. Bigger (2010). 18.12544/011: "Mine-Blast Loading: Experiments and Simulations," Southwest Research Institute. Herndon : US Army Tank-Automotive Research, Development and Engineering Center,

Appendix Saturation Ratio

The saturation ratio is the fraction of pores that are filled with water, often expressed as

$$S_w = \frac{V_w}{V_v} \qquad (5)$$

where V_w is the volume of water and V_v is the volume of voids. Typically the volume of water is determined by weighing the sample and then oven drying the sample to determine M_w the mass of water. The volume of water is calculated via

$$V_w = \frac{M_w}{\rho_w} \qquad (6)$$

where $\rho_w = 1000\ \text{kg/m}^3$. The void volume is obtained by subtracting the soil particle volume from the total volume

$$V_v = V - \frac{M_s}{\rho_s} = V - \frac{M_s}{G\rho_w} \qquad (7)$$

Where M_s is the mass of the dry soil and ρ_s is the grain density often take to be 2640 kg/m^3. The second form uses the specific gravity of the soil particles $G = \rho_s / \rho_w$.

Now the saturation ratio can be expressed as

$$S_w = \frac{V_w}{V_v} = \frac{M_w / \rho_w}{V - M_s / \rho_s} \qquad (8)$$

Noting the volume can be expressed in terms of the sample density and mass

$$V = \frac{M}{\rho} = \frac{M_w + M_s}{\rho} \qquad (9)$$

The saturation ratio can be rewritten as

$$\begin{aligned} S_w &= \frac{V_w}{V_v} = \frac{M_w / \rho_w}{\left(M_w + M_s\right)/\rho - M_s / \rho_s} = \frac{\frac{\rho_s}{\rho_w} M_w}{\frac{\rho_s}{\rho}\left(M_w + M_s\right) - M_s} \\ &= \frac{\frac{\rho_s}{\rho_w}\frac{M_w}{M_s}}{\frac{\rho_s}{\rho}\left(\frac{M_w}{M_s} + 1\right) - 1} = \frac{G\omega}{\frac{\rho_s}{\rho}\left(1 + \omega\right) - 1} \end{aligned} \qquad \textbf{(10)}$$

where the notation $\omega = M_w / M_s$ i.e. *gravimetric water content,* has been introduced.

Appendix Dry Density [3]

Starting from the total mass

$$M = M_w + M_s \qquad (11)$$

rearranging

$$M = M_s\left(1+\omega\right) \qquad (12)$$

where $\omega = M_w / M_s$ is the gravimetric water content, and finally

$$M_s = \frac{M}{\left(1+\omega\right)} \qquad (13)$$

The definition of the dry density is

$$\rho_d = \frac{M_s}{V} = \frac{M}{V\left(1+\omega\right)} = \frac{\rho}{\left(1+\omega\right)} \qquad (14)$$

Appendix – Two Spring LS-DYNA Input Template

```
*Keyword
$
$        C:\All-Mine\LS-DYNA\Version971\ls-dyna_smp_d_Dev_115126_winx64.exe
$                         i=Two-Springs.k
$
$*******************************************************************************
$                           Len Schwer
$            Schwer Engineering & Consulting Services
$                         Len@Schwer.net
$                       Revised  4 July 2017
$*******************************************************************************
$
$  UNITS:
$                    length -> millimeters
$                    time   -> milliseconds
$                    mass   -> grams
$                    force  -> Newtons
$                    stress -> F/A = Newtons/mm^2 = MPa
$
$ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
$   Input:
$           GrainD  -- grain density typically 2640kg/m^3
$           Density -- density of tested soil, i.e. wet density
$
$   Output:
$           NODOUT -- Node 1 X-Displacement
```

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[3] Derivation provided by Dr. Lance Besaw.

```
$           SPCFOR -- Node 2 X-Force
$ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
$
$
*TITLE
  Spring Model for Partially Saturated Soil
$
*Parameter
$       1         2         3         4         5         6         7
r Tstart   0.0      r Tend      5.00    i States     20     r Tmax     1.00e-4
r TSSFAC   0.6      i LCTM      500     i Binary     1
$
$   PORO -- Porosity = 1 - dry density / grain density
$   GrainD -- grain density typically 2640kg/m^3
$   H20Den -- water density typically 1000 kg/m^3
$   Density -- density of tested soil, i.e. wet density
$   H2OPer -- water percentage --  w = mass of water / mass of soilds
$
r GrainD   2.641E-3 r H2ODen   1.0E-3
r DenDry   1.219E-3
$r Density  1.342E-3 r H2OPer    0.10
r Density  1.525E-3 r H2OPer    0.25
$r Density  1.770E-3 r H2OPer    0.45
$r Density  1.770E-3 r H2OPer    0.45
$r Density  1.750E-3 r H2OPer    0.45
$
*Parameter_Expression
$
 r TDplot    (Tend-Tstart)/(States-2)
 r TASCII    TDplot/100.0
 r TLBE       4.0*TDplot
 r TASCITR   TDplot/100.0
$
 r Tend2     2.0*Tend
$
r PORO       1.0 - DenDry/GrainD
r SpecG      GrainD/H2ODen
$
$
r RhoRat     GrainD/Density
r SatRat     SpecG*H2OPer/(RhoRat*(1 + H2OPer) - 1.0)
r VoidR      PORO*(1.0 - SatRat)
$
$
r VStrain     log(1.0 - VoidR)
$
r Compres     (-1.0*VStrain) + 0.2
r SprLen      (-1.0*VStrain) + 0.2
$
$========1=========2=========3=========4=========5=========6=========7=======
$                                Control CARDS
$========1=========2=========3=========4=========5=========6=========7=======
$
$*Control_Structured_Term
$
*CONTROL_CONTACT
0.900,0.000E+00,0,0,0,0,0,0
0,0,0,0,0.000E+00,0,0,0
*CONTROL_ENERGY
2,0,0,0
```

```
*CONTROL_OUTPUT
$ NPOUT  NEECHO  NREFUP  IACCOP  OPIFS  IPNINT  IKEDIT  IFLUSH
     1,    3,     0,        0,     0.0,    0,    99000,    0
$  IPRTF IERODE TET10S8 MSGMAX IPCURV GMDT IP1DBLT EOCS
      ,      ,      ,      ,     0
*CONTROL_SOLUTION
$ SOLN  NLQ  ISNAN  LCINT
   0,    0,   1
*CONTROL_TERMINATION
&Tend,0,0.000E+00,0.000E+00,25.0
$
*CONTROL_TIMESTEP
$ DTINIT TSSFAC ISDO TSLIMT DT2MS  LCTM ERODE MS1ST
  0.0,   0.90,    0, 0.0,   0.0,  &LCTM,   0,    0
$
$=======1=========2=========3=========4=========5=========6=========7=======
$                               Database CARDS
$=======1=========2=========3=========4=========5=========6=========7=======
$
*DATABASE_ELOUT
&TASCII, &Binary
*DATABASE_GLSTAT
&TASCII, &Binary
*DATABASE_JNTFORC
&TASCII, &Binary
*DATABASE_MATSUM
&TASCII, &Binary
*DATABASE_NODOUT
&TASCII, &Binary
*DATABASE_RBDOUT
&TASCII, &Binary
*DATABASE_RCFORC
&TASCII, &Binary
*DATABASE_SLEOUT
&TASCII, &Binary
$
*DATABASE_DEFORC
&TASCII, &Binary
*DATABASE_SPCFORC
&TASCII, &Binary
$
*DATABASE_BINARY_D3PLOT
&TDplot
$
*DATABASE_EXTENT_BINARY
$ NEIPH NEIPS MAXINT STRFLG SIGFLG EPSFLG RLTFLG ENGFLG
   8,    14,    0,      1,     1,     1,     0,    0
$ CMPFLG IEVERP BEAMIP DCOMP SHGE STSSZ N3THDT IALEMAT
   0,     0,      0,     0,    0,   0
$ NINTSLD PKP_SEN SCLP HYDRO MSSCL THERM INTOUT NODOUT
    1
$
*DATABASE_HISTORY_NODE
$  ID1 ID2 ID3 ID4 ID5 ID6 ID7 ID8
    1,  2
$
*Comment *DATABASE_HISTORY_BEAM
$  ID1 ID2 ID3 ID4 ID5 ID6 ID7 ID8
  1,  2,  3
$=======1=========2=========3=========4=========5=========6=========7=======
```

```
$                             NODE & ELEMENT CARDS
$========1=========2=========3=========4=========5=========6=========7=======
$
*Node
1,     0.0, 0.0, 0.0
2, &SprLen, 0.0, 0.0
$
*Element_Discrete
$ EID  PID  N1  N2  VID   S   PF   OFFSET
   1,  100,  1,  2,     ,   ,   0
   2,  200,  1,  2,     ,   ,   0
$
*Set_Node_List
$  SID
   12
$ NID1  NID2  NID3
   1,     2
$
*ELEMENT_MASS_Node_Set
$    eid     nid            mass     pid
     321      12          1.0E-3
$
$========1=========2=========3=========4=========5=========6=========7=======
$                             SECTION CARDS
$========1=========2=========3=========4=========5=========6=========7=======
$
*Section_Discrete
$  SECID DRO KD V0 CL FD
    100, 0
$  CDL TDL
   0.0, 0.0
$
$========1=========2=========3=========4=========5=========6=========7=======
$                             PART CARDS
$========1=========2=========3=========4=========5=========6=========7=======
$
*Part
       Soil Spring
$  PID  SECID  MID EOSID HGID GRAV ADPOPT TMID
   100,  100,  100
$
*Part
        Water Spring
$  PID  SECID  MID EOSID HGID GRAV ADPOPT TMID
   200,  100,  200
$
$
$========1=========2=========3=========4=========5=========6=========7=======
$                             BOUNDARY SPC CARDS
$========1=========2=========3=========4=========5=========6=========7=======
$
*BOUNDARY_SPC_Node
$  NID/NSID CID DOFX DOFY DOFZ DOFRX DOFRY DOFRZ
      1,     0,  0,    1,    1,     1,    1,    1
      2,     0,  1,    1,    1,     1,    1,    1
$
$========1=========2=========3=========4=========5=========6=========7=======
$                             MATERIAL CARDS
$========1=========2=========3=========4=========5=========6=========7=======
$
```

```
*MAT_SPRING_NONLINEAR_ELASTIC
$  MID LCD LCR
   100, 100
$
*MAT_SPRING_NONLINEAR_ELASTIC
$  MID LCD LCR
   200, 200
$
$========1=========2=========3=========4=========5=========6=========7=======
$                              DEFINE CURVE CARDS
$========1=========2=========3=========4=========5=========6=========7=======
$
*DEFINE_CURVE
$  LCID SIDR   SFA   SFO OFFA OFFO DATTYP LCINT
   100,     , 1.0, 1.0
$  Displacement   Force               Soil 10% H2O
-0.6216,  -49.7094
-0.6211,  -49.6443
-0.6184,  -48.6499
-0.6156,  -47.6154
-0.6128,  -46.6062
-0.6099,  -45.5696
-0.6070,  -44.5981
-0.6040,  -43.5279
-0.6011,  -42.5334
-0.5983,  -41.5765
-0.5954,  -40.9492
-0.5918,  -39.6900
-0.5885,  -38.7228
-0.5850,  -37.7700
-0.5813,  -36.7753
-0.5777,  -35.7534
-0.5738,  -34.8091
-0.5699,  -33.9614
-0.5652,  -32.6370
-0.5605,  -31.5353
-0.5559,  -30.5384
-0.5508,  -29.4261
-0.5455,  -28.3453
-0.5400,  -27.2266
-0.5347,  -26.1814
-0.5292,  -25.1466
-0.5234,  -24.1223
-0.5173,  -23.1022
-0.5110,  -22.0925
-0.5043,  -21.0597
-0.4972,  -20.0269
-0.4897,  -19.0192
-0.4818,  -17.9842
-0.4733,  -16.9094
-0.4642,  -15.9394
-0.4545,  -14.8519
-0.4439,  -13.8460
-0.4325,  -12.8108
-0.4206,  -11.7735
-0.4092,  -10.9039
-0.3953,  -9.8665
-0.3826,  -9.0286
-0.3668,  -8.0628
-0.3489,  -7.1998
```

```
-0.3297,  -6.2755
-0.3156,  -5.6856
-0.3013,  -5.0849
-0.2805,  -4.3942
-0.2523,  -3.5561
-0.2171,  -2.6927
-0.1742,  -1.9102
-0.1332,  -1.3235
-0.0966,  -0.8862
-0.0635,  -0.6032
-0.0403,  -0.3654
-0.0272,  -0.2740
-0.0182,  -0.2410
-0.0136,  -0.2158
-0.0116,  -0.1528
-0.0097,  -0.1323
-0.0076,  -0.0756
-0.0044,  -0.0535
-0.0012,  -0.0409
0.0000,  -0.0425
   1.0,  0.00
$
*DEFINE_CURVE
$  LCID SIDR  SFA  SFO          OFFA   OFFO DATTYP LCINT
   200,     , 1.0, 1.0,    &VStrain
$  Displacement   Force               Water
-0.218,  -1098.580
-0.202,  -948.338
-0.186,  -815.029
-0.170,  -697.454
-0.154,  -592.631
-0.138,  -499.765
-0.123,  -416.641
-0.107,  -342.727
-0.091,  -276.353
-0.076,  -217.157
-0.061,  -163.861
-0.045,  -116.222
-0.030,  -73.245
-0.015,  -34.763
  0.0,    0.0
 -&VStrain, 1.0E-6
   1.0,   0.0
$
$
*DEFINE_CURVE
$  LCID SIDR SFA SFO OFFA OFFO DATTYP LCINT
   400
$  Time  Displacement                  Loading
   0.0, 0.0,
  &Tend, &Compres
 &Tend2, &Compres
$  &Tend, &PORO
$ &Tend2, &PORO
$
$
*DEFINE_CURVE
$  LCID SIDR SFA SFO OFFA OFFO DATTYP LCINT
   &LCTM
$  Time  Time Step                  Time Step
```

```
    0.0, 0.01
 &Tend2, 0.01
$
$========1=========2=========3=========4=========5=========6=========7=======
$                             NODAL LOAD CARDS
$========1=========2=========3=========4=========5=========6=========7=======
$
*BOUNDARY_PRESCRIBED_MOTION_Node
$  typeID DOF VAD LCID SF VID DEATH BIRTH
     1,    1,  2,  400
$
$
$
*End
```