Batted-Ball Performance of a Composite Softball Bat as a Function of Ball Type

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Abstract

*Experimental and finite element methods were used to model the collision between a composite softball bat and softballs of different COR (Coefficient of Restitution) and compression specifications. Experimental bat characterization methods included barrel compression and modal analysis. Experimental softball characterization methods included COR, CCOR (Cylindrical Coefficient of Restitution), compression and dynamic stiffness. Finite element models were built in HyperMesh and analyzed in LS-DYNA®. Softballs were modeled using material models #6, #57 and #83, and the composite softball bat was constructed according to the manufacturer's specifications using *PART_COMPOSITE. Three methods to calibrate the finite element softball models were investigated and included "flat-surface" and "cylindrical-surface" coefficients of restitution and DMA (dynamic mechanical analysis). The "cylindrical-surface" test was found to be the most effective method of calibration to predict the batted-ball speed (BBS) as measured in bat/ball impact testing.*

Introduction

In the early 2000s, composite bats gained broad attention in the softball community when some bat manufacturers produced bats that significantly outperformed the best aluminum bats on the market. One composite bat performed so well that the ASA (Amateur Softball Association) quickly banned it and threatened to ban all composite bats as this governing body had done with the titanium bats in 1993. However, before taking such drastic action, the ASA revisited its process for imposing limits on the performance of softball bats [1]. Today, bats used in ASA championship play must be certified to be in compliance with performance regulations at an ASA-Approved testing facility [2]. However, even with restrictions that place aluminum bats and composite bats on a "level playing field" with respect to batted-ball performance in the certification test, composite bats have become the bat of choice for a majority of softball players.

Finite element (FE) modeling has been used in sports ball simulations such as cricket, baseball, tennis and golf [3-6]. A good finite element model that can accurately predict bat-ball performance is an extremely valuable tool to assist in the design of a bat, as it reduces the need for labor intensive, time consuming and expensive experimental prototyping and testing. Particularly with composite layups, any number of combinations of materials and ply angles can be explored using the model, which would otherwise take weeks to explore using a prototype testing program.

A credible finite element model of the ball-bat collision for softball is challenging. Achieving such a model is difficult primarily because of variations in the processing of the softballs' polyurethane cores which can yield different properties of the overall ball, e.g. hardness and liveliness, and the response of the softball during a bat-ball collision is rate dependent. The mechanical behavior of the composite bat is slightly less challenging to model because the bat material can be assumed to be essentially linear elastic unless significant material damage is induced during the collision. The ideal models should have the flexibility to allow for changes in bat and ball constructions and have the ability to capture how BBS varies as a result of these changes. If such models were available, then one could customize the bat design with the goal to maximize the BBS for a given ball construction.

This paper presents a summary of the complimentary experimental and finite element studies that were completed to develop a bat-ball collision model for the research of composite softball bats. Softballs were characterized using simple tests, and finite element models of the softballs were calibrated to yield good correlation to the experimental characterization tests. The calibrated softball models were then used to explore their ability to correlate with bat-ball collisions using a composite softball bat.

Experimental Methods

Two types of softballs and one composite bat were investigated using experimental methods. Softball characterization data were used to find material constants for finite element models of the softballs. The composite bat data were used to demonstrate the credibility of the finite element model of a composite softball bat that was built using ply layup information from the bat manufacturer. Bat performance data were collected for subsequent comparison to finite element simulations of bat/ball impacts to explore how well independent calibration of bat and ball models could work as a predictive design tool.

Ball Compression Testing

Testing for softball compression was conducted in accordance to ASTM F1888-09 [7]. Softballs were compressed between two flat surfaces and oriented such that the plates contacted between the seams as shown in Figure 1 to a displacement of 0.25 in. at a rate of 1 in/min. The force required to displace the ball the prescribed distance was recorded, the ball was rotated 90°, and compressed again. The average of the two forces was recorded as the compression load for the softball.

Figure 1. Softball compression test setup.

Ball COR/CCOR and Dynamic Stiffness Testing

Testing was conducted to determine the COR, CCOR and the DS of the softballs using the test setup shown in Figure 2. The flat impact surface used in the testing is a 3 x 4 in. x 1-in. thick steel plate, and the cylindrical impact surface is a 2.63-in. diameter half cylinder. The impact plate is mounted on three piezoelectric load cells that measure the forces during impact at a rate of 10^5 samples per second. A ball carrier called a “sabot” shown in Figure 3 is used to fire the ball and ensures the ball impacts the plate between the seams and without spin. The sabot impacts the arrestor plate, and the ball travels through the three light gates.

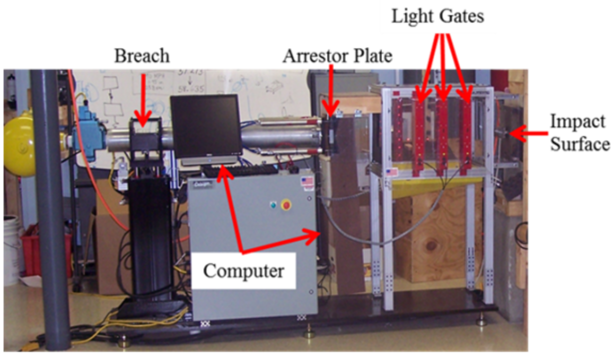

Figure 2. Dynamic stiffness test setup.

Figure 3. Softball loaded in a ball carrier, or "sabot."

Bat Barrel Compression

Bat barrel compression testing was done to quantify the stiffness of the barrel section of the softball bat. The test setup used is shown in Figure 4. The bat is placed in the fixture and compressed between two 3.86-in. diameter steel half-cylinder platens at the 6-in. location as measured from the endcap. The hand crank shown in the figure rotates to displace the bottom platen and compress the barrel.

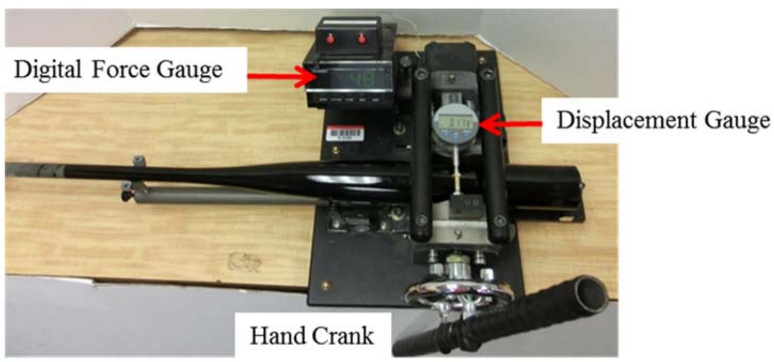

Figure 4. Barrel compression test setup.

First, an initial preload of 5-15 lb. is applied to the barrel, and both gauges are zeroed to establish the reference, or zero displacement point. The barrel is then compressed 0.02 in., the force is recorded, and the gauges are zeroed again. The barrel is compressed an additional 0.05 in. where the force is measured a final time. The force required to displace the barrel the first 0.02 in. is referred to as the "preload" and the force to compress it the final 0.05 in. is referred to as the "final load". The bat is then rotated a prescribed amount, and the process is repeated. The barrel compression is the average of the final loads at the 0°, 90°, 45° and -45° orientations.

Modal Analysis

Impact modal analysis was done on the composite softball bat to determine the first two bending and hoop frequencies. The bat was suspended from the ceiling using two strings to simulate a free-free condition as shown in Figure 5. Two accelerometers were attached approximately 90° from each other 1 in. from the endcap as shown in Figure 5.

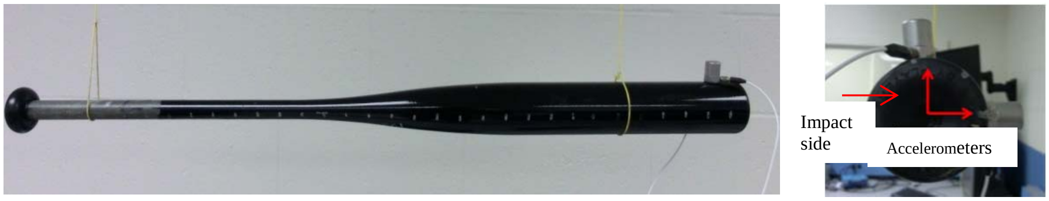

Figure 5. Modal test setup of a simulated free-free condition with accelerometers attached 90° from each other to measure the bending and hoop frequencies of the bat.

The bat was impacted with a plastic-tip force hammer, on the opposite side of one of the accelerometers as shown in Figure 5. The impulse and accelerometer data were collected by a Dactron Photon II four-channel FFT analyzer, and the RT Pro Photon data acquisition program analyzed the frequency response. The frequency range of 1000 Hz with 800 spectral lines of resolution was used to determine the first two bending modes, and the range was increased to 2000 Hz to determine the first two hoop modes. The frequency response function (FRF) was computed from the time response using the Fast Fourier transform based on the ratio of acceleration output to force input [8].

The natural frequencies were determined using a "peak pick" technique on the FRF response function. The bending frequencies are the first peaks to occur, and the hoop frequencies are indicated when the response functions from each of the accelerometers overlay each other with the same magnitude. A typical FRF for a hollow baseball bat showing the first three bending and first two hoop modes is shown in Figure 6 [8].

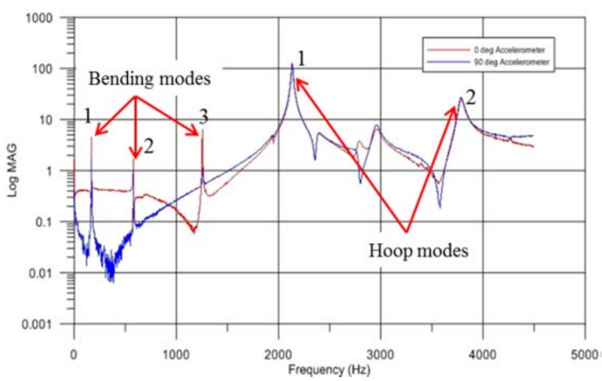

Figure 6. FRF measurement from a hollow baseball bat, displaying three bending modes and two hoop modes [8].

Batted-Ball Speed

The batted-ball speed (BBS) is found in a controlled lab test and is used to determine the relative performance of the different softballs on the composite softball bat. The air cannon shown in Figure 7 was used for this testing. The bat is held stationary at the start of the test and is able to pivot about the 6-in. location as measured from the base of the knob. The ball is fired at the barrel 6 in. from the endcap, and the inbound and outbound velocities are measured by three light gates. Testing was performed at 95 and 110 mph. Balls are fired using a sabot in the same configuration as the dynamic stiffness testing. The cannon was controlled by a LabVIEW program on the computer shown in Figure 7, and the program displays the inbound and outbound velocities. The inbound and outbound velocities are used to calculate the BBS of the bat and ball combination.

Figure 7. LVS sports cannon configuration.

Modeling

This section discusses the methods used for the finite element models of the softballs and bats for analysis in LS-DYNA. Models were built in HyperMesh v11.0 and analyzed in LS-DYNA version 971 R6.0.0 using double precision. It was found that using flat wall COR data to calibrate the material parameters of the ball did not translate as well as CCOR calibration for predicting batted-ball speeds. Thus, only the CCOR ball modeling is reported in this paper.

Construction of the Softball Finite Element Model

Softballs were modeled as an isotropic, homogeneous sphere with 12096 eight-noded solid elements and used Material Models #6 General Viscoelastic, #57 Low-Density Urethane Foam, and #83 Fu-Chang Foam with Rate Effects. The default constant-stress reduced-integration solid elements were used. LS-DYNA notes that when large deformation is seen in the elements, such as in a CCOR test or bat-ball collision, the one-point reduced integrated elements are more robust than the fully integrated twenty-noded solid elements and can reduce hourglassing in the model [9].

CCOR Models

The cylindrical impact surface for the CCOR model was meshed with 2664 solid elements. The target is defined as a rigid material using Material Model #20, and all nodes are constrained in translation and rotation in the material card. An example CCOR model with boundary conditions and the global axis used are shown in Figure 8. The explicit solver was used for completing the impact analyses.

Ball Compression Model

The ball compression models were conducted using implicit analysis due to the relatively slow speed at which compression tests are performed. The compression plates are defined as rigid bodies with Material Model #20 and meshed with 2000 elements each as shown in Figure 9. The bottom plate (blue) is fixed in translation and rotation, and the top plate is given a prescribed velocity of 0.02 in/sec. The time period for the analysis is 14 sec., and the compression value of the ball is determined by the force associated with a 0.25-in. displacement. The contact option

*AUTOMATIC_SURFACE_TO_SURFACE was used to define the interaction between the ball and the plates.

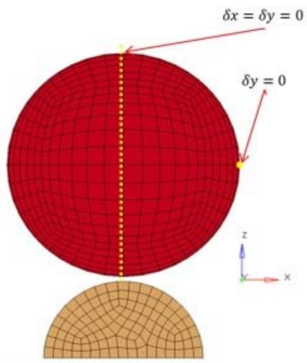

Figure 8. Example CCOR model with boundary conditions.

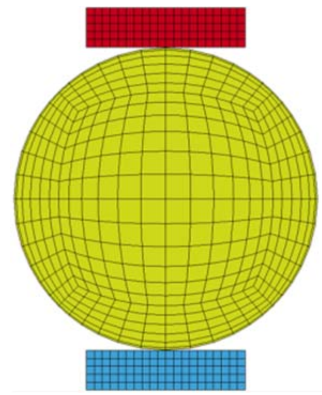

Figure 9. Softball compression test. The bottom (blue) plate is fixed, while the top (red) plate is given a prescribed velocity to compress the ball.

Construction of the Composite Bat Finite Element Model

For the current research, the authors were fortunate to have the manufacturer provide the details of the ply layup for a double-wall composite softball bat. The composite softball bat model was built with 20494 four-noded thin shell elements. *PART_COMPOSITE was used to define the angle, thickness and material of each layer in the laminate. The bat is comprised of braided layers of glass and carbon fibers. Each braided ply is defined in the model using a [+/-/+/-/+] configuration, where the "+" layers each have 1/6th of the total thickness of the ply, and each "-" layer has 1/4th of the total thickness of the ply. Defining each braid layer of the bat in this manner eliminates the bending/extensional coupling that occurs in a non-symmetrical laminate stack-up [10]. The bat was partitioned into ten components including the knob and endcap because the number of plies and angles vary along the length of the bat due to the differences in diameter in the handle, taper and barrel sections. The knob and endcap are fixed to the handle and barrel, respectively, with merged coincident nodes.

Material model #22 was used for each layer of the laminate where the density, Poisson's ratios, Young's Moduli, and shear moduli are defined. The local material axis was defined using the AOPT = 3 option for orthotropic materials. The material axis option allows the model to account for the change in the orientation of the surface normal with respect the global reference frame due to the cylindrical shape of the bat.

In the case for thin shell elements as used in this research, the normal is always perpendicular to the plane of the element. The orientation angle for any given ply of the laminate was defined locally for each element using the *ELEMENT_SHELL_BETA card which is a rotation in the plane of the element from the vector $a = v \times n$.

The "double-wall" part of the bat refers to a 10-in. long tube (insert) that is pressed into the barrel of the bat, with a 0.001-in. interference fit. The contact definition between the barrel and insert was defined as *INTERFERENCE_SURFACE_TO_SURFACE contact. Dynamic relaxation (DR) was used to introduce the pre-stressed state of the interference fit in the barrel section. This method uses a "pseudo" analysis that is performed quasi-statically either implicitly or explicitly before the transient analysis. During the pseudo analysis, the interference is gradually eliminated by scaling the contact thickness from zero to a maximum value of unity [11]. The interface stiffness is scaled from zero to unity during the DR phase with a load curve

(LCID1) defined in the contact definition, and this stiffness is held constant during the transient phase with a second load curve (LCID2) [12].

During the pseudo analysis, the ratio of current to peak kinetic distortional energy is monitored, such that the pseudo phase terminates when the ratio falls below the user-defined convergence tolerance (DRTOL) [12]. This stressed state then becomes the initial state for the subsequent transient analysis. The DR phase can be seen graphically in the binary output "d3drlf" file as shown in Figure 10. At the conclusion of the DR phase, a prescribed geometry file (drdisp.sif) is written. This file is utilized by setting IDRFLAG = 2 in subsequent analyses to invoke the prestressed state without having to repeat the DR convergence [12].

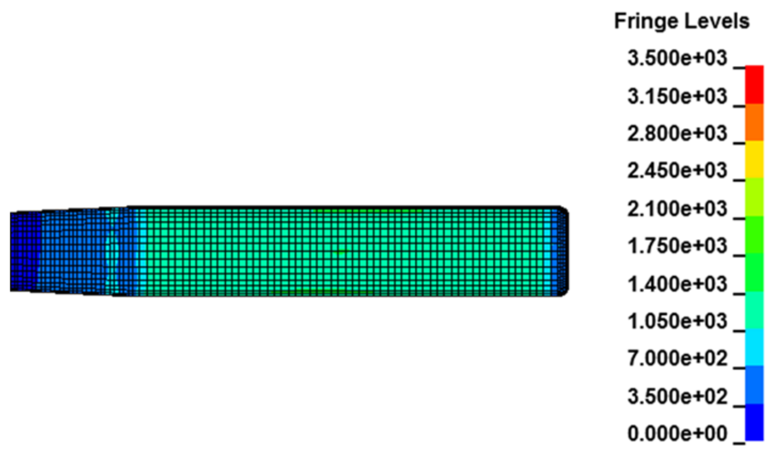

Figure 10. Example von Mises stress fringe plot during the dynamic relaxation pseudo phase for the barrel of the bat. Stress units are psi.

Barrel Compression

The barrel compression model was solved using the implicit solver. As shown in Figure 11, the barrel was modeled between two rigid half-cylinder platens each with a diameter of 3.3 in. and meshed with 8960 solid elements. The bottom platen was fixed for translation and rotation. The top platen was only permitted to translate in the y-direction and was given a prescribed velocity of 3.5×10^{-3} in/sec to compress the barrel 6 in. from the endcap.

The circumferential nodes on the barrel and insert were constrained so as not to allow translation in the x-direction, and the four nodes that lie on the y-axis were constrained in the z-direction to ensure the bat does not slip while it is being compressed. The boundary conditions are shown in Figure 11 and Figure 12.

The *AUTOMATIC_SURFACE_TO_SURFACE contact was used to define the interaction between the platens and barrel. The force to displace the barrel was extraxcted from the ascii file rcforc, and the displacement of the barrel was determined from the binary time history plot. The time period for the analysis was 25 sec., and the force vs. displacement was plotted to determine the barrel compression. The interfacial forces between the inside of the barrel and the insert were also of interest. The insert was meshed into two components as shown in Figure 13 to observe the forces from the "top" of the barrel and the "bottom" of the barrel separately.

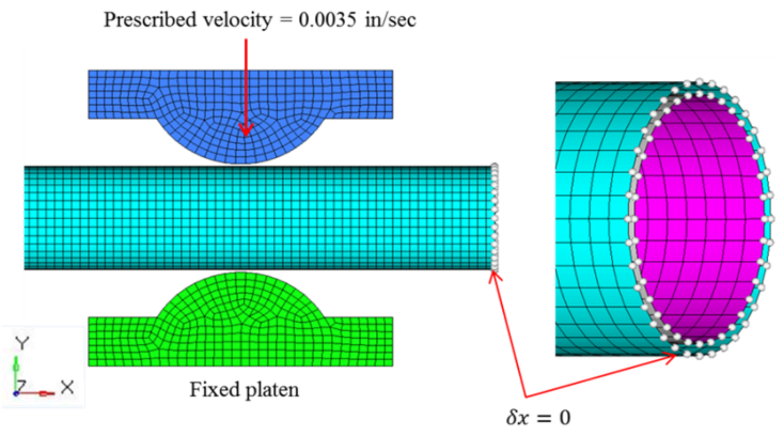

Figure 11. Barrel compression model configuration with boundary conditions (x-y view).

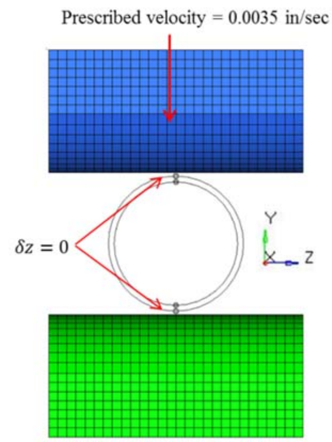

Figure 12. Barrel compression model configuration with boundary conditions (y-z view).

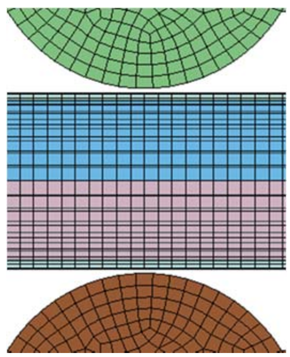

Figure 13. Insert inside barrel meshed into two components.

Modal Analysis Model

A free-free modal analysis was completed using LS-DYNA as a method to calibrate the finite element model of the softball bat model to correlate with the physical bat. The goal was to correlate the first two bending and hoop modes. The analysis was completed using the implicit solver. Thirty modes were extracted to ensure the first and second bending and first and second hoop modes were found during the analysis.

Batted-Ball Performance Model

In the batted-ball performance testing model, the composite bat was supported by an aluminum pivot which was meshed with 17891 reduced-integration solid elements as shown in Figure 14. The *AUTOMATIC_SURFACE_TO_SURFACE contact definition was used to define the interaction between the pivot and the bat. The pivot has a single node measured 6 in. from the base of the knob that is fixed in translation and rotation. This boundary condition keeps the bat stationary until it is impacted with the ball, then allows for rotation after the impact. The ball was given an initial velocity when fired at the bat. The contact between the ball and bat was also defined as *AUTOMATIC_SURFACE_TO_SURFACE. The time period for the modeling of the bat-ball impact was 0.002 sec. to ensure the ball has reached a steady-state velocity after the impact.

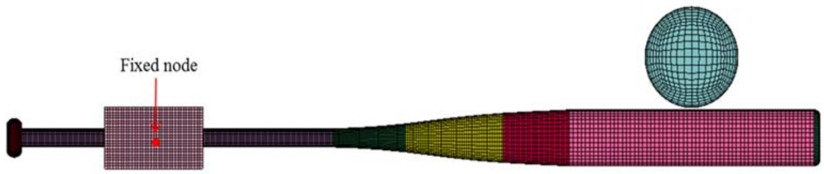

Figure 14. Bat-ball finite element models for a composite softball bat.

Ball Material Model

A softball is made of a polyurethane core wrapped in a leather cover. As a result, the ball behavior is viscoelastic where the deformation and energy dissipation are a function of the rate and state of deformation. A number of the viscoelastic material models that are available within LS-DYNA were explored for the modeling of the ball. The material models considered included #6 Viscoelastic, #57 Low-Density Urethane Foam, and #83 Fu-Chang Foam with Rate Effects. For this research, Material Model #6, often called the Power Law model [13], was found to give the best correlation with experimental data.

Results

Physical Properties

Table 1 compares the experimental and finite element results for the weight, MOI (Mass Moment of Inertia) and Balance Point of the 33.1-in. long composite bat. Each of the categories shows good correlation between the model and experimental values. The MOI shows a 1.4% difference from experiment.

Table 1. Experimental and finite element results for bat physical properties.

Weight (oz.)		MOI ($oz\text{-}in^2$)		Balance Point (in. from base of the knob)	
Exp.	FE	Exp.	FE	Exp.	FE
22.1	22.1	6920	7020	21.0	21.0

Barrel Compression

Table 2 lists the experimental and finite element barrel compression results for the composite softball bat with the insert. The preload is the force associated with the first 0.02-in. displacement, and the final load is the force associated with the additional 0.05-in. displacement. After the preload is applied, the force is "zeroed" out and is presented this way in Table 2. The experimental values listed are the averages of the compression values at the four barrel orientations.

Table 2. Experimental and finite element composite barrel compression results

Method	Preload (lbs.)	Final Load (lbs.)
Experiment	186	368
FE	155	386

The bat finite element model shows excellent correlation to experimental data, as the final loads are within 5% between the model and the experiment. The finite element model of the bat can be justified to have a higher barrel stiffness than the experiment because the model assumes perfect bonding between plies, and thus, does not consider flaws or any variations in ply orientations which can occur during manufacturing [10]. Even without this rationalization for the slight difference between the experimental and finite element compression values, this correlation is remarkable and infers that the finite element model of the bat is an excellent representation of the physical bat.

<u>Modal Analysis</u>

Table 3 shows the experimental and finite element model results of the free-free modal analysis. The first two bending and hoop modes are compared. All modes show reasonably good correlation between the model and the experimental data. Because the finite element model of the bat showed such good agreement to the experimental MOI and BP, no adjustments were made to the mass distribution to assist in improving the correlation to the bending frequencies. The stiffness of the handle was also shown to influence the bending frequencies, however to preserve the manufacturer's composite layup specifications, the material definitions were not altered.

Table 3. Composite softball bat experimental and finite element free-free modal analysis results

Method	1st Bending (Hz)	2nd Bending (Hz)	1st Hoop (Hz)	2nd Hoop (Hz)
Experiment	146	555	1450	1950
FE	126	468	1406	2086

<u>Ball Calibration: "Cylindrical-Surface" Calibration</u>

Two softball types were used in the experimental CCOR/DS testing and are likewise referenced as Ball Types 1 and 2. Material model #6 parameters as determined through parametric studies for each ball model are given in Table 4. Table 5 outlines the experimental and FE results for Ball Types 1 and 2 CCOR testing at 95 and 110 mph. The percent difference from experiment is also shown. Table 4 shows the experimental and FE results for Ball Types 1 and 2 DS testing at 95 and 110 mph. The results indicate the DS is underestimated by the FE model, with the Ball Type 1 model showing a 14% difference at 95 mph, and an 8% difference at 110 mph. The Ball Type 2 model showed a 20% difference at 95 mph and a 14% difference at 110 mph. The peak loads and contact times were also compared, and the results are shown in Tables 7 and 8. The experimental and FE force vs. time curves are also compared in Figure 15. The FE ball model shows better agreement to the peak loads than the DS. The model also shows reasonable correlation to the contact time seen in experiment. The results in Table 8 show that experimental contact time for Ball Type 2 is approximately 0.2 ms longer than Ball Type 1, which is likely a result of the lower compression and the lower dynamic stiffness of Ball Type 2. It is recommended that an "automated" optimization study be pursued in the near future to explore the ability to achieve a better correlation between the model results and the experimental data.

Table 4. Material model #6 parameters used for cylindrical surface calibration for ball types 1 and 2.

Ball Type	K (psi)	G_0 (psi)	G_∞ (psi)	β
1	800000	22500	750	75000
2	100000	11000	640	60000

Table 5. Ball types 1 and 2 experimental and FE results for CCOR at 95- and 110-mph

Ball Type	95-mph CCOR				110-mph CCOR			
	Exp.	FE		% Difference	Exp.	FE		% Difference
		Material Model	CCOR			Material Model	CCOR	
1	0.380	6	0.381	0.26	0.355	6	0.368	3.66
2	0.460	6	0.456	0.87	0.442	6	0.439	0.68

Table 6. Experimental and FE results for DS at 95- and 110-mph

Ball Type	95-mph DS				110-mph DS			
	Exp. (lb/in)	FE		% Difference	Exp. (lb/in)	FE		% Difference
		Material Model	DS (lb/in)			Material Model	DS (lb/in)	
1	7190	6	6153	14.40	7260	6	6680	8.06
2	5760	6	4610	20.0	5780	6	4980	13.76

Table 7. Experimental and FE results for peak loads at 95- and 110-mph

Ball Type	95-mph CCOR Peak Load				110-mph CCOR Peak Load			
	Exp. (lbs.)	FE		% Difference	Exp. (lbs.)	FE		% Difference
		Material Model	Load (lbs.)			Material Model	Load (lbs.)	
1	4700	6	4390	6.36	5540	6	5300	4.46
2	4150	6	3690	11.05	4780	6	4440	7.00

Table 8. Experimental and FE results for contact time at 95- and 110-mph

Ball Type	95-mph CCOR Contact Time				110-mph CCOR Contact Time			
	Exp. (ms)	FE		% Difference	Exp.	FE		% Difference
		Material Model	Time (ms)			Material Model	Time (ms)	
1	1.03	6	1.20	16.50	1.01	6	1.09	7.92
2	1.21	6	1.40	16.50	1.20	6	1.30	8.33

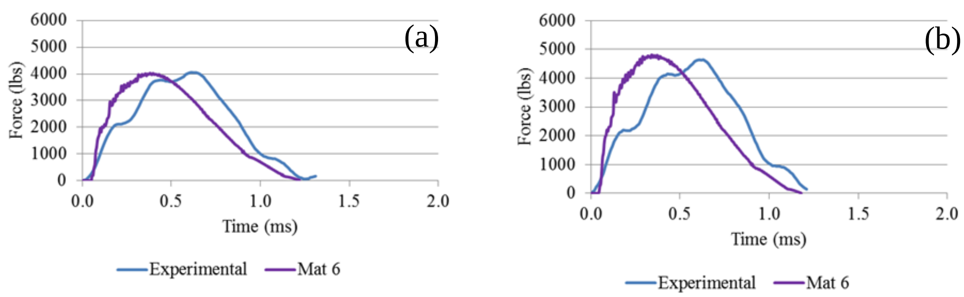

Figure 15. Ball Type 2 force vs. time. (a) 95-mph CCOR and (b) 110-mph CCOR.

Batted-Ball Speed Using Calibrated Softballs and Composite Bat

Next, the "cylindrical-surface" calibrated softball models were fired at the 6-in. location on the composite softball bat. The BBS results are listed in Table 9. The experimental results for Ball Type 1 showed a slight increase (0.50 mph) in BBS at the higher test speed for impacts with the composite bat. Ball Type 2 shows relatively good correlation to the BBS but underestimated the BBS by 5% at 95 mph and 3% at 110 mph. Also, the experimental results for Ball Type 2 showed a decrease of 2.06 mph from the 95 to 110 mph test speed. This decrease was able to be captured by Material model #6, while on a smaller scale of 0.045 mph. This slight reduction for the model is not shown in the table due to rounding of the speeds to the nearest 0.1 mph.

Table 9. Ball types 1 and 2 experimental and FE performance results for impacts with the composite bat at 95 mph and 110 mph

Ball Type	95-mph BBS				110-mph BBS			
	Exp. (mph)	FE		% Difference	Exp. (mph)	FE		% Difference
		Material Model	BBS (mph)			Material Model	BBS (mph)	
1	96.5	6	95.8	0.74	97.0	6	97.2	0.37
2	95.2	6	90.1	5.40	93.2	6	90.1	3.35

Conclusions

Experimental characterization of a softball bat and softballs of different COR and compression specifications were conducted. Complementary finite element models were constructed in HyperMesh, analyzed in LS-DYNA and were manually tuned to give reasonable correlation with the experimental test data. The modal and barrel compression behaviors of the bat were studied. Reasonable correlation was achieved for bat ball collisions. However, future work to use a more sophisticated method to conclude the material parameters is recommended.

Acknowledgement

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