Interaction Methods for the SPH Parts (Multiphase Flows, Solid Bodies) in LS-DYNA®

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Abstract

The interesting and complex behaviour of fluids emerges mainly from interaction processes. Smooth particles hydrodynamics is a meshfree, Lagrangian particle method and a simple, yet flexible method for modeling fluid flows and solid bodies in a robust way. It has been applied extensively to the multiphase flows, heat conduction, high explosive problems and so on. In this paper, different interaction methods available in the LS-DYNA for SPH parts which have wide range of density and material properties are studied and compared. Node to node contacts fit well for the interaction between two SPH parts with high density ratio, the standard SPH interpolation method has better accuracy around the interfaces when two SPH parts have similar density and material properties. Different interaction approaches can be combined together in one model to reach the best results. Also the interactions between Lagrangian elements with SPH particles are discussed. Some examples are presented to show how to use different approaches with different combination of LS-DYNA keywords.

Introduction

SPH is a Lagrangian method for solving partial differential equations. Essentially, the domain is discretized by approximating it by a series of roughly equi-spaced particles. They move and change their properties (such as temperature) in accordance with a set of ordinary differential equations derived from the original governing PDEs. SPH was first applied by Lucy (1977) to astrophysical problems, and was extended by Gingold (1982). Cloutman (1991) used SPH to model hypervelocity impacts. Libersky and Petschk have shown that SPH can be used to model materials with strength. In recent years it has been developed as a method for incompressible isothermal enclosed flows by Monaghan (1994).

As a Lagrangian method, the interaction between SPH particles and FEM elements can be easily handled by a normal node to surface contact in LS-DYNA. Because of its ability to handle large distortions by avoiding the need for intensive FEM remeshing, its reasonable precision and stability compared with classical methods FEM, SPH is a competitive approach compared to finite elements (FE) and is increasingly being used in some fast-transient dynamics problems. Several authors have proposed to couple FE and SPH which seems a reasonable approach in order to benefit from the advantages of both formulations. In LS-DYNA, hybrid elements that enable coupling effects between SPH particles and FEM solid are implemented. In this method, hybrid elements are configured to facilitate coupling effect of solid element and smoothed particle hydrodynamics (SPH). This method can be used to adaptively transform a Lagrangian solid Part or Part Set to SPH particles. Also hybrid elements are defined in a computer aided engineering (CAE) grid model as a buffer or interface between the SPH particles and FEM solids

When simulating fluids, it is important to capture interaction effects accurately in order to reproduce real world behavior. Smoothed Particle Hydrodynamics has shown to be a simple, yet flexible method to cope with many fluid simulation problems in a robust way. Unfortunately, the results obtained when using SPH to simulate miscible fluids are severely affected, especially if density ratios become large. In SPH, particles have a spatial distance covered by smooth length over which their properties are smoothed by a kernel function. Problems arise when rest densities and masses of neighboring particles vary within the smoothing length, as in such cases the smoothed quantities of a particle show falsified values. The undesirable effects reach from unphysical density and pressure variations to spurious and unnatural interface tensions, as well as severe numerical instabilities. A node to node penalty based contact was introduced to avoid those interface effects in LS-DYNA.

We have couples of keyword options available in LS-DYNA for the interactions between SPH parts also the interactions between SPH part and Solid part: Standard SPH interpolation method (normal way); Node to node penalty based contact through keyword *DEFINE_SPH_TO_SPH_COUPLING; Combination of both method through keyword *SECTION_SPH_INTERACTION; Node to surface contact between SPH part and Solid part; Coupling between SPH parts and solid parts through keyword *DEFINE_ADAPTIVE_SOLID_TO_SPH; Coupling between SPH particles with ALE elements through keyword *ALE_COUPLING_NODAL_PENALTY. Different interaction approaches can be combined together in one model to reach the best results. Some examples are demonstrated to show how to use different approaches with different combination of LS-DYNA keywords.

Standard SPH formulation

Fundamentals of the SPH method

Particles methods are based on quadrature formulas on moving particles $(x_i(t), w_i(t)) i \in P$, P is the set of the particles. $x_i(t)$ is the location of particle i and $w_i(t)$ is the weight of the particle i. The quadrature formulation for a function can be written as:

$$\int_{\Omega} f(x)dx = \sum_{j \in P} w_j(t) f(x_j(t)) \qquad (1)$$

The quadrature formulation (1) together with the definition of smoothing kernel leads to the definition of the particle approximation of a function. The interpolated value of a function:
$u(X)$ at position X
using the SPH method is:

$$\prod^h (u(x_i)) = \sum_{j \in \Omega} w_j(t) u(x_j) W(x_i - x_j, h) \qquad (2)$$

Where the sum is over all particles inside Ω and within a radius $2h$, W is a spline based interpolation kernel of radius $2h$. It mimics the shape of a delta function but without the infinite tails. It is a C^2 function. The kernel function is defined as following:

$$W(x_i - x_j, h) = \frac{1}{h}\theta\left\{\frac{x_i - x_j}{h(x,y)}\right\} \qquad (3)$$

$W(x_i - x_j, h) \rightarrow \delta$ when $h \rightarrow 0$, δ is Dirac function, h is a function of x_i and x_j and is the so-called smoothing length of the kernel.

And the cubic B-spline function is defined:

$$\theta(d) = C \times \begin{cases} 1 - \frac{3}{2}d^2 + \frac{3}{4}d^3 & \text{when } 0 \le d \le 1 \\ \frac{1}{4}(2-d)^3 & \text{when } 1 \le d \le 2 \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

The gradient of the function $u(X)$ is given by applying the operator of derivation on the smoothing length:

$$\nabla \prod^h (u(x_i)) = \sum_j w_j u(x_j) \nabla W(x_i - x_j, h) \tag{5}$$

Evaluating an interpolated product of two functions is given by the product of their interpolated values.

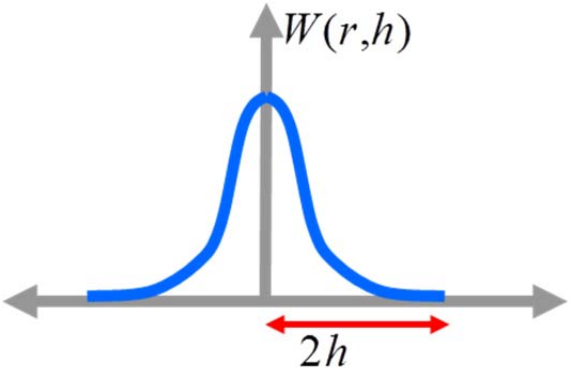


Fig 1. Support size of 2d kennel function

Continuity equation and Momentum equation

The particle approximation of continuity equation is defined as:

$$\frac{d\rho_i}{dt} = \rho_i \sum_j \frac{m_j}{\rho_j} \left(v^{\beta}{}_i - v^{\beta}{}_j\right) W_{ij'\beta} \tag{6}$$

It is Galilean invariant due to that the positions and velocities appear only as differences, and has good numerical conservation properties. ${v^{\beta}}_i$ is the velocity component at particle i.

The discretized form of the SPH momentum equation is developed as:

$$\frac{dv_i^{\alpha}}{dt} = -\sum_j \frac{m_j}{\rho_i \rho_j} (\sigma_i^{\alpha\beta} \pm \sigma_j^{\alpha\beta}) W_{ij,\beta} \tag{7}$$

The above formulation ensures that stress is automatically continuous across material interfaces. Different types of SPH momentum equations can be achieved through applying the identity equations into the normal SPH momentum equation. Symmetric formulation of SPH momentum equation can reduce the errors arising from particle inconsistency problem.

From equation (7), the following particle body forces were derived:

$$\begin{aligned} F_i^{pressure} &= -\sum_j m_j \frac{p_i + p_j}{2\rho_j} \nabla W(r_{ij}, h) \\ F_i^{vis\cos ity} &= \mu \sum_j m_j \frac{v_i - v_j}{2\rho_j} \nabla^2 W(r_{ij}, h) \end{aligned} \tag{8}$$

Where $r_{ij} = x_i - x_j$, μ is the viscosity coefficient of the fluid. The pressure p_i are computed via the constitutive equation:

$$p_i = k(\rho_i - \rho_0) \tag{9}$$

where k is the stiffness of the fluid and ρ_0 is its initial density.
Finally, for the acceleration of a particle i, we have:

$$\mathbf{a}_i = 1/\rho_i (F_i^{pressure} + F_i^{vis\cos ity} + F_i^{external}) \tag{10}$$

Where $F_i^{external}$ are external forces such as body forces or forces due to contacts.

Multiple Fluids and Solid bodies

The above equations (1)--(10) were designed to handle single phase fluid and can be easily extended in order to handle multiple fluids and Solid bodies with different rest density. Cares must be taken to avoid the interface instability due to the large density ratio across the interfaces.

Interaction through standard SPH interpolation

As shown in Fig 2, the standard way to handle the interactions between different SPH parts is through the SPH interpolation functions (i.e treated as one part for multiple SPH fluids) and no contact treatments are needed on the interfaces of the different SPH parts. In SPH, particles have a spatial distance (smoothing length) over which their properties are smoothed by a kernel function (such as density, pressure). Smoothed quantities of a particles show falsified values when densities and masses of neighboring particles vary largely within the smoothing length. As shown in Muller et al (2005), miscible fluids with a density ratio larger than 10 cannot be realistically simulated if the standard SPH density summation is used. The reason is that in SPH, the macroscopic flow is mainly governed by the density computation. Over or underestimating the density leads to erroneous pressure values, which might result in unnatural acceleration caused by erroneously introduce pressure ratio (Ihmsen et al 2011). Also lead to a spurious interface tension and a large gap between the fluids. The erroneous quantities lead to undesirable effects, reaching from unphysical density and pressure variations to spurious and unnatural interface tensions, and even to severe numerical instabilities

Another issue with the interaction through standard SPH interpolation is that different SPH fluid parts may stick together after the interaction due to the SPH function interpolations. To activate this option, CONT parameter in *CONTROL_SPH has to be set as 0, and no contacts are allowed between those SPH parts.

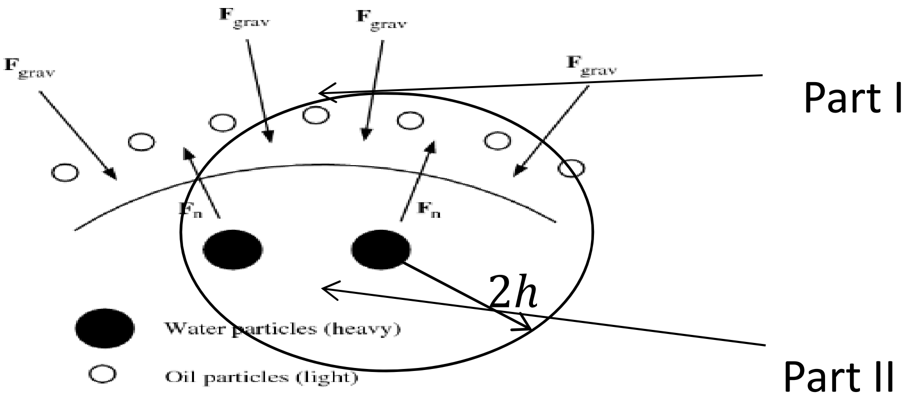


Fig 2. Interaction through SPH interpolation (treated as one part and no contact is needed)

Interaction through node to node contacts

A penalty based node to node contact model is introduced on the interfaces of the different SPH parts. As shown in Fig 3, all the SPH interpolations (density, pressure and so on) are carried out inside the local domains of each SPH part. No spurious interface tension or interfaces instability happened in this model. The contact forces on the interfaces will be applied to the external forces as in equation (10).

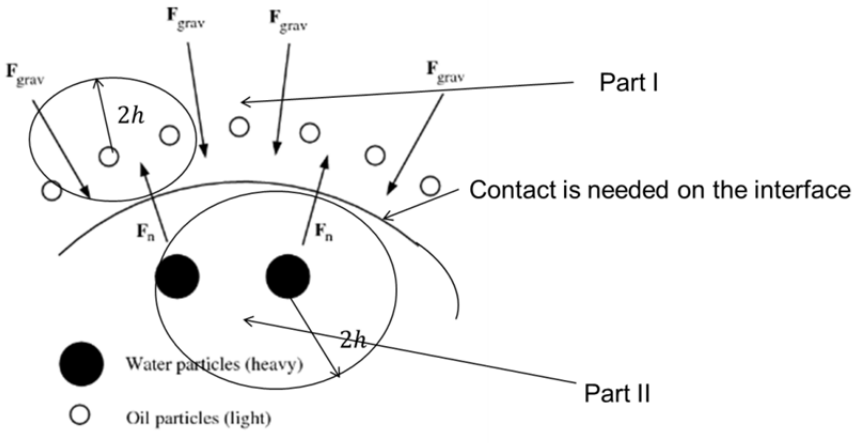


Fig 3. Interaction through node to node contacts

In this system, the repulsive contact force acting on particle due to contact F_c , is directly proportional to the displacement or overlap between particles δ:

$$F_c = K_l\delta \tag{11}$$

where $\delta = \mathrm{d} - 2\mathrm{h}$ and K_l is the linear-spring constant or stiffness. If the contact is modeled using only this linear-spring, no energy will be consumed and the contact will be ferfectly elastic. In reality, some kinetic energy is dissipated in plastic deformation, and/or converted to heat or sound energy. To account for those energy losses, a contact damping force based on a dashpot model is also included:

$$F_d = \eta v \tag{12}$$

The contact damping force is proportional to the relative velocity of the contacting particles, where the constant of proportionality η is known as the damping coefficient, $v = v_1 - v_2$.

A *DEFINE_SPH_TO_SPH_COUPLING keyword is needed between any two SPH parts for the contact interaction, also parameter CONT in *CONTROL_SPH need to be set as 1 to deactivate the interaction through standard interpolation.

Interaction through both normal interpolation method and node to node contact method in one model

Combine the CONT=1 option in *CONTROL_SPH keyword with keyword *SECTION_SPH_INTERACTION to support the partial interactions between SPH parts through normal interpolation option and partial interaction between SPH parts through node to node contacts in one model. All the SPH parts that are defined with *SECTION_SPH_INTERACTION keyword will integrate with each other through normal interpolation method automatically, and node to node contacts are needed for the interactions between SPH parts defined with *SECTION_SPH_INTERACTION keyword and other SPH parts, also for the interactions between any other SPH parts that not defined through *SECTION_SPH_INTERACTION keyword.

Normally, interaction through the standard interpolation method produces more consistent results across the interface when SPH parts interacted have similar densities and material properties, however the smoothed quantities of a particles show falsified values when densities and masses of neighbouring particles vary largely within the smoothing length. The erroneous quantities lead to undesirable effects, reaching from unphysical density and pressure variations to spurious and unnatural interface tensions, and even to severe numerical instabilities. Interaction through node to node contact allow the users to select the desired amount of contact force between two SPH parts by choosing the desired penalty scale factors according to the simulation problem at hand and it help to avoid the instabilities due to large density ratios at the interfaces. Also for any two SPH parts with total Lagrangian formulation definition, only node to node contact can be used as interaction method, since the neighbouring lists are only updated at beginning for SPH particles with total Lagrangian formulation definition. With *SECTION_SPH_INTERACTON keyword, users can take advantage of both interaction methods in one model based on the SPH parts' properties as shown in Fig 4.

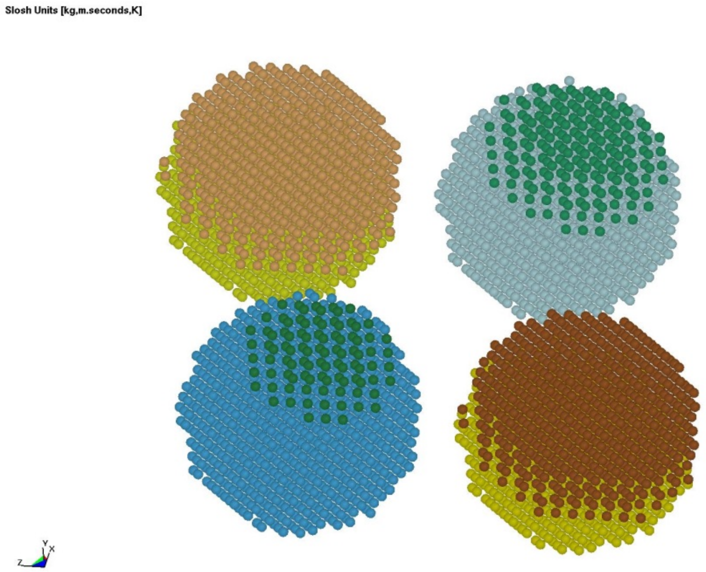


Fig 4. Interaction through both standard interpolation methods and contact methods

SPH coupling with Solid elements

For the normal interaction between SPH particles and FEA elements (Solids and Shells), node to surface contacts in LS-DYNA can be used, since both methods are based on the Lagrangian description. When modelling SPH particles as fluids flow and FEA elements as structure in the models, the Fluid Structure Interaction problems can be easily handled by the node to surface contact.

Keyword *DEFINE_ADAPTIVE_SOLID_TO_SPH is used to adaptively transform a Lagrangian solid Part or Part Set to SPH particles, when the Lagrangian solid elements comprising those parts fail (Shown in Fig 5). One or more SPH particles (elements) will be generated for each failed element. The SPH particles replacing the failed element inherit all of the properties of the failed solid element, e.g. mass, kinematic variables, and constitutive properties.

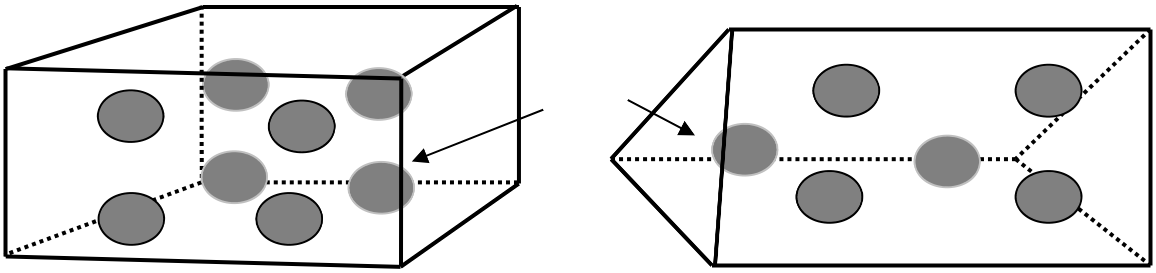


Fig 5. Transform Solid elements into SPH particles

With ICPL=0, this keyword is used for debris simulation, no coupling happens between newly generated SPH particles and solid elements, user need to define node to surface contact for the interaction between those two parts. When ICPL=1 and IOPT=1, the newly generated SPH particles are bonded with solid elements as one part through the coupling (Hybrid elements).

With ICPL=1 and IOPT=0, this keyword is used as Hybrid Element coupling SPH with Solid (as shown in Fig 6).

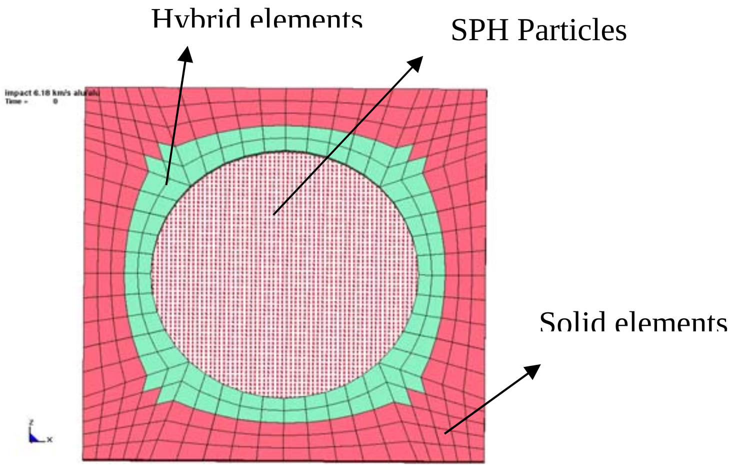


Fig 6. Example of Hybrid elements a stransit layers between SPH particles and Solid elements

In this hybrid elements method, we have the SPH formulation which can endure quite large deformation and at the same time we have the Solid meshes which clearly describe the material interfaces. Solid elements constrain SPH nodal locations. SPH elements provide “penalty force” against solid nodal motions. Hybrid elements are used as transit layers between SPH elements and Solid elements, for a portion of grid model comprises SPH particles because the likelihood of enduring large deformation, while the rest of the model comprises FEM solid elements, hybrid elements are placed between the solids and the particles, each hybrid element comprises two layers: solid layer and particle layer.

SPH coupling with ALE, DEM methods

Keyword *ALE_COUPLING_NODAL_PENALTY provides a penalty base contact interaction between SPH particles and ALE materials (master segments). Also keyword *DEFINE_SPH_DE_COUPLING defines a penalty based contact between SPH particles and DEM particles. This option uses the node to node contacts to couple SPH solver with discrete element sphere (DES) solver.

Examples

Water, air impacting with rigid ring

3D tank with fluids which has the dimension of 1.0X0.8X0.01 (Fig. 7) was calculated to validate the node to node contact in LS-DYNA for multiple SPH parts with high density ratio across the interfaces. The fluids in the tank were water and air with air on the top, the density ratio between those two fluids is more than 1000. Both water and air were model with SPH particles. A rigid ring modeled with cylinder shell impacted the fluids in the tank with the speed of 50 in Y direction. The results from the SPH particles were compared with the results from the ALE method with the same dimension and parameters (see Fig.9 and Fig.10).

In the model, automatic_ node_ to_surface contacts were used for the interaction between air, water particles and rigid shells, a node to node contact was used for the interaction between air particles and water particles. The contact between two SPH particles from different parts was detected when the distance of two particles is less than SRAD*(sum of smooth lengths from two particles)/2.0. SRAD is parameter ranged from 0 to 1.0 and is used to adjust the detecting criteria due to initial penetration.

The standard interaction through SPH interpolation will not work for this case. A proper penalty scale factor has to be used for better performance. As show in Fig. 8, a double value of penalty scale factor will cause more noises around the interface of the two SPH fluids. The final deformed shape of water was comparable with the results from ALE elements (Fig. 9). The velocity historys for the rigid ring from both SPH model and ALE model were plotted and compared in Fig. 10, two results were close.

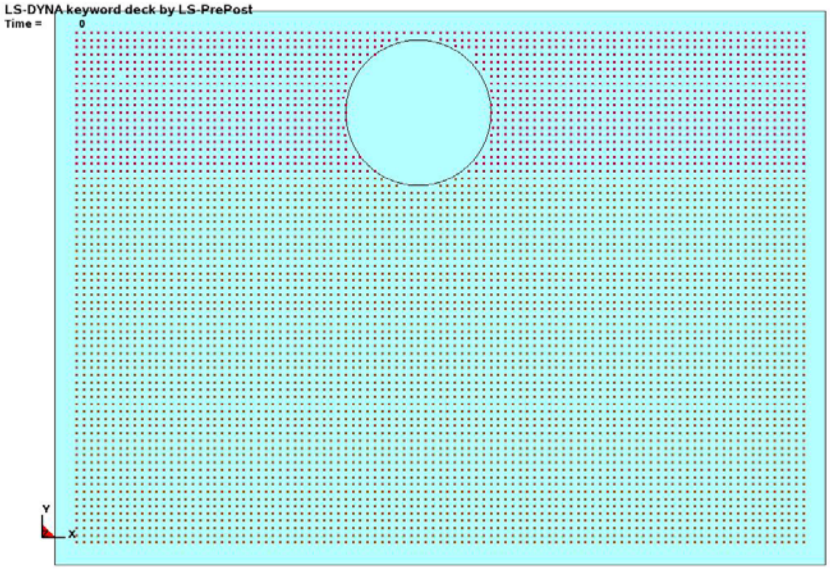
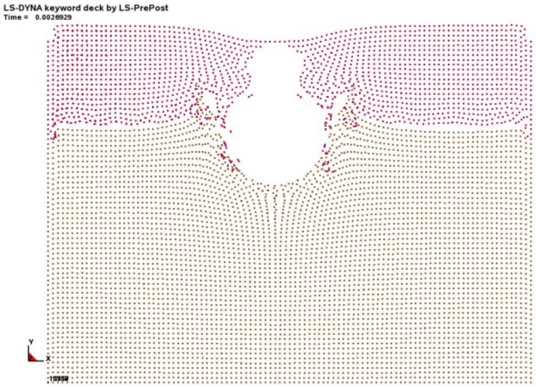
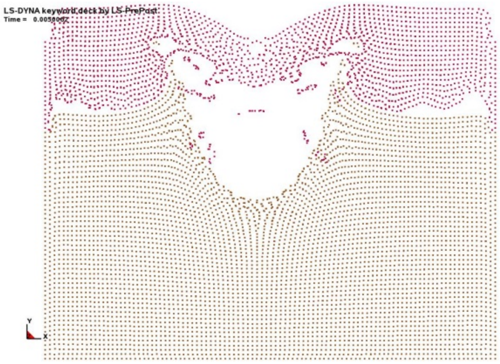


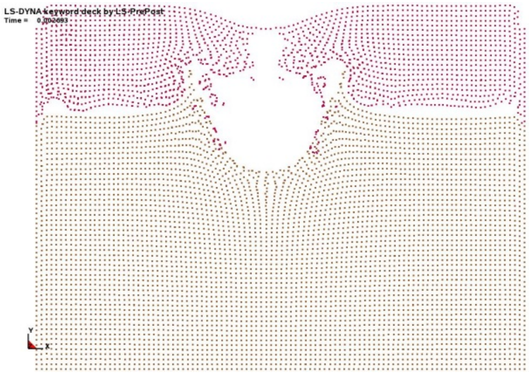
Fig 7. Problem set up of water impact



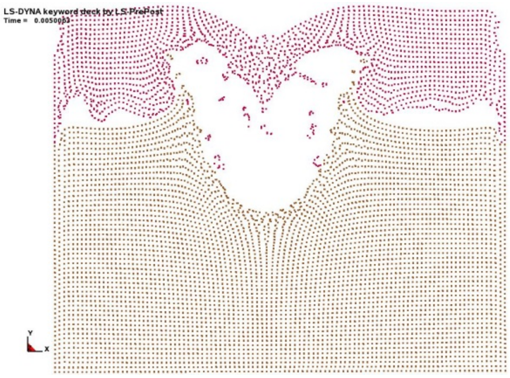
t=2.7 ms



t=5.0 ms



t=2.7 ms



t=5.0 ms

Fig 8. Upper: deformation shape for air and water model
Lower: deformation shape with double value of penalty scale factor

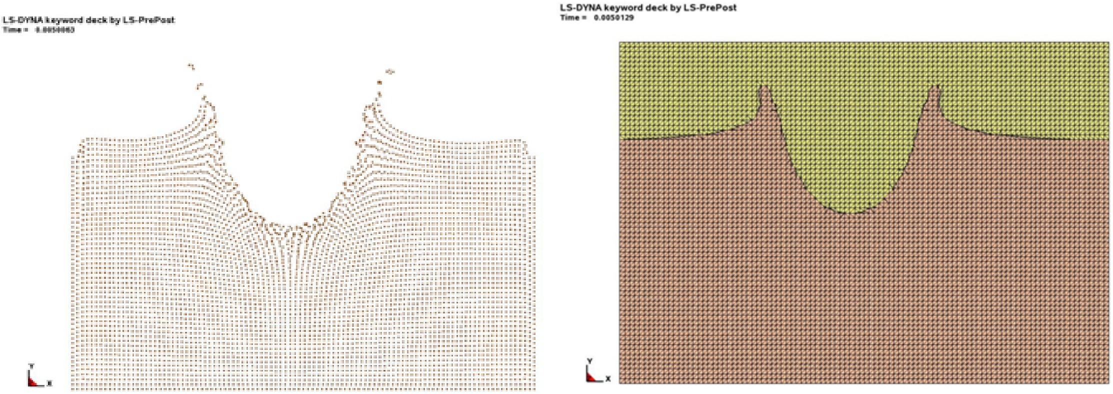


Fig 9. Final deformation shape from SPH model (left) compared to ALE model (right)

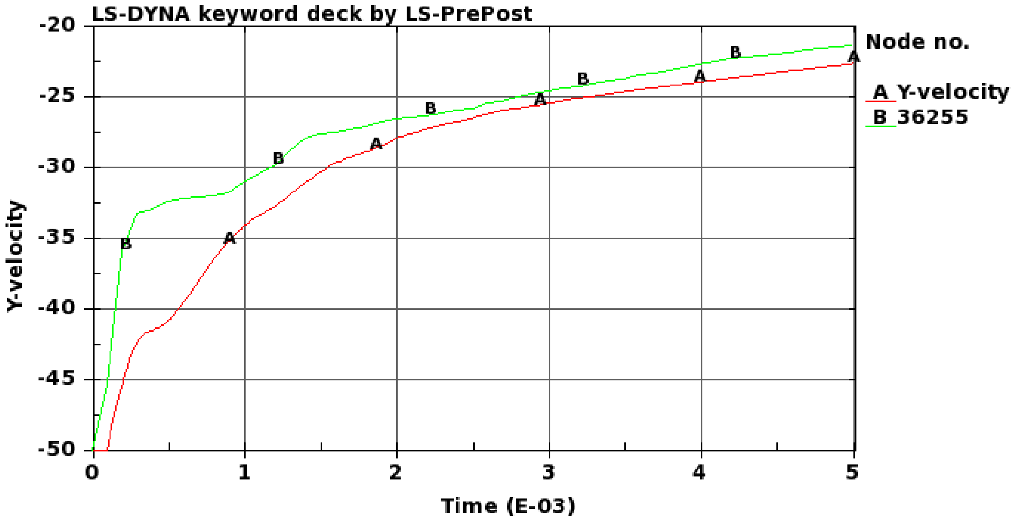


Fig 10. Impact velocity from SPH model (B) compared to velocity from ALE model (A)

Summary

We present couples of keyword options available in LS-DYNA for the interactions between SPH parts also the interactions between SPH part and Solid part: Standard SPH interpolation method (normal way); Node to node penalty based contact through keyword *DEFINE_SPH_TO_SPH_COUPLING; Combination of both method through keyword *SECTION_SPH_INTERACTION; Node to suface contact between SPH part and Solid part; Coupling between SPH parts and solid parts through keyword *DEFINE_ADAPTIVE_SOLID_TO_SPH; Coupling between SPH particles with ALE elements through keyword *ALE_COUPLING_NODAL_PENALTY. Different interaction approaches can be combined together in one model to reach the best results. Some examples are demonstrated to show how to use different approaches with different combination of LS-DYNA keywords.

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