Determining the Material Constants for Mullins Effect in Rubber Part One: Uniaxial

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Introduction

In this paper, the strain-energy density with Mullins damage function on unloading and subsequent reloading is considered. We introduce a damage function that has four material constants: two for unloading and two for subsequent reloading. The effect of these constants on unloading and subsequent reloading is studied for uniaxial extension. We determine these four material constants from a set of numerically generated uniaxial extension test data. The mathematical formulation has been implemented in LS-DYNA® for user application and evaluation.

This paper will be extended to two-dimensional problems and a set of biaxial test data will be obtained and analyzed. The second part of this paper will be presented in another LS-DYNA conference.

Formulation

The strain-energy density function with Mullins damage function of a rubber is $\tilde{W}(\lambda_i)$, and

$$\tilde{W}(\lambda_i) = \eta W(\lambda_i) \tag{1}$$

where W is the strain-energy density function based on the initial loading, and $\eta = \eta(W)$ is a damage function for the Mullins effect.

$$\frac{\partial \tilde{W}}{\partial \lambda_i} = \eta \frac{\partial W}{\partial \lambda_i} + W \frac{\partial \eta}{\partial \lambda_i} = \left(\eta + W \frac{\partial \eta}{\partial W} \right) \frac{\partial W}{\partial \lambda_i} \tag{2}$$

The Cauchy stresses (force per unit deformed area) are

$$t_1 = \frac{1}{\lambda_2 \lambda_3} \frac{\partial \tilde{W}}{\partial \lambda_1} \tag{3}$$

There are two similar equations for t_2 and t_3.

The following damage function, a Cauchy first-order ordinary-differential equation, is chosen for this report.

For initial loading $$W\frac{\partial \eta}{\partial W}+\eta=1 \tag{4a}$$

For unloading $$W\frac{\partial \eta}{\partial W}+\eta=1-\frac{1}{r_1}\tanh\left[\frac{1}{m_1}\left(1-\frac{W}{W_m}\right)\right] \tag{4b}$$

For subsequent reloading $$W\frac{\partial \eta}{\partial W}+\eta=1-\frac{1}{r_2}\tanh\left[\frac{1}{m_2}\left(1-\frac{W}{W_m}\right)\right] \tag{4c}$$

$W_m(\lambda_i)$ is the maximum strain-energy density function before unloading. r_1, r_2, m_1 and m_2 are the material constants for the Mullins damage function. With this damage function the unloading and subsequent reloading follow different paths, as shown in Figure 1. For a loading with a value of the strain-energy density function greater than $W_m(\lambda_i)$, the process repeats.

For Mooney-Rivlin materials the strain-energy density equation is:

$$W=C_1(I_1-3)+C_2(I_2-3)=C_1[(I_1-3)+\alpha(I_2-3)] \tag{5}$$

where C_1 and C_2 are material constants and $\alpha=C_2/C_1$. The strain invariants I_1 and I_2 are written in terms of the principal stretch ratios λ_1, λ_2 and λ_3

$$\begin{aligned} I_1&=\lambda_1^2+\lambda_2^2+\lambda_3^2\\ I_2&=\lambda_1^2\lambda_2^2+\lambda_2^2\lambda_3^2+\lambda_3^2\lambda_1^2 \end{aligned} \tag{6}$$

An incompressibility condition is assumed in the Mooney-Rivlin material constitutive equation, so that

$$\lambda_1\lambda_2\lambda_3=1 \tag{7}$$

For uniaxial tension or compression in 1-direction, $\lambda_2=\lambda_3$ and $\lambda_2^2=\lambda_1^{-1}$
Hence,

$$\begin{aligned} I_1&=\lambda_1^2+2/\lambda_1\\ I_2&=2\lambda_1+1/\lambda_1^2 \end{aligned} \tag{8}$$

Cauchy stress t_1 is related to the uniaxial stretch ratio λ_1.

For initial loading $$t_1 = 2C_1\left(\lambda_1^2 - \frac{1}{\lambda_1}\right)\left(1 + \frac{\alpha}{\lambda_1}\right) \quad (9a)$$

For unloading $$t_1 = 2C_1\left\{1 - \frac{1}{r_1}\tanh\left[\frac{1}{m_1}\left(1 - \frac{W}{W_m}\right)\right]\right\}\left(\lambda_1^2 - \frac{1}{\lambda_1}\right)\left(1 + \frac{\alpha}{\lambda_1}\right) \quad (9b)$$

For subsequent reloading $$t_1 = 2C_1\left\{1 - \frac{1}{r_2}\tanh\left[\frac{1}{m_2}\left(1 - \frac{W}{W_m}\right)\right]\right\}\left(\lambda_1^2 - \frac{1}{\lambda_1}\right)\left(1 + \frac{\alpha}{\lambda_1}\right) \quad (9c)$$

The result for uniaxial extension obtained from EXCEL calculations is shown in Figure 1. The material constants are:

$C_1 = 50$, $\alpha = 0.1$, $r_1 = 0.8$, $m_1 = 1.0$, $r_2 = 0.5$ and $m_2 = 5$.

The material is first stretched to point (1) followed by the initial loading path. After point (1) the material is unloaded to the unstretched state and then reloaded to point (1) again. The stress-stretch ratio curves are shown in the figure and the Mullins effect is clearly seen. The material is then further stretched past point (1) to point (2). No Mullins effect occurs after point (1). The material is then unloaded again to the origin and reloaded again to point (2). The Mullins effect is seen again.

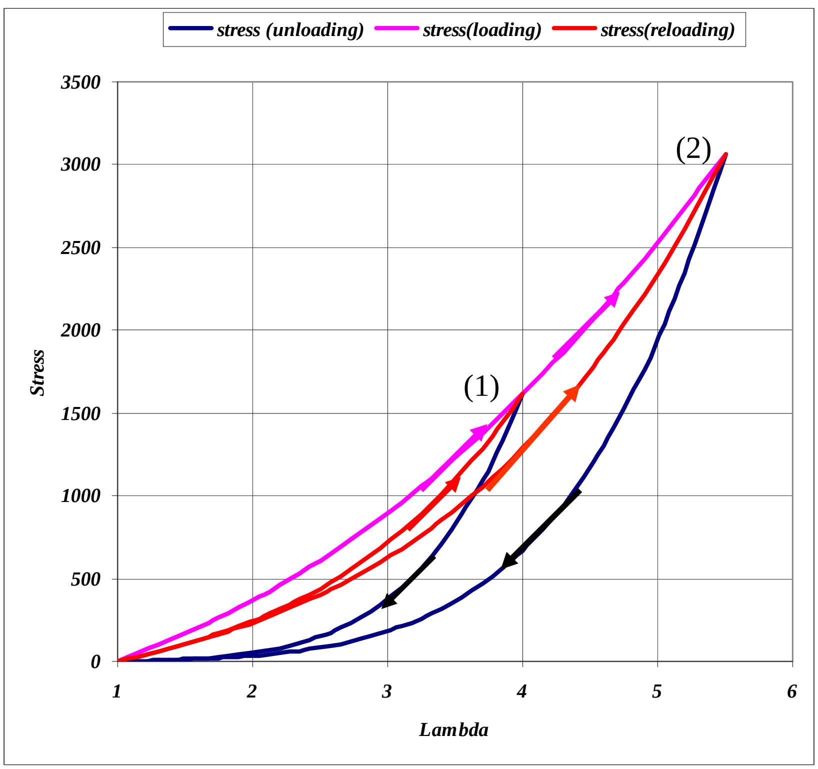

Figure 1. The Mullins effect for uniaxial loading, unloading and subsequent reloading, obtained from EXCEL.

The effects of constants r_1 and m_1

The effect of constants r_1 and m_1 on unloading is studied for uniaxial extension. The effect of r_1 is shown in Figure 2. The effect of m_1 is shown in Figure 3. They cover a wide range of unloading values. When either r_1 or m_1 approaches a large value, then

$$\frac{1}{r_1}\tanh\left[\frac{1}{m_1}\left(1-\frac{W}{W_m}\right)\right]\to 0\,,\ \text{and}\ \ W\frac{\partial\eta}{\partial W}+\eta\to 1\,. \tag{10}$$

The loading and unloading curves coincide and the Mullins effect vanishes.

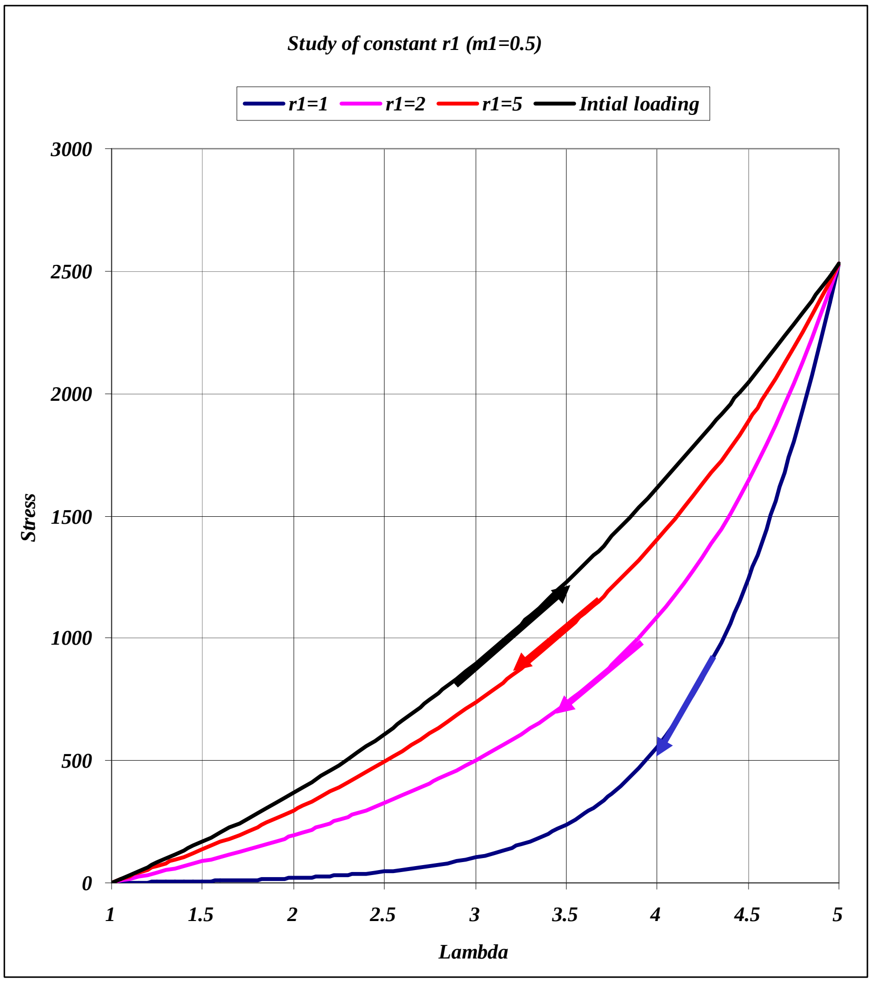

Figure 2. The effect of r_1 on unloading for uniaxial extension

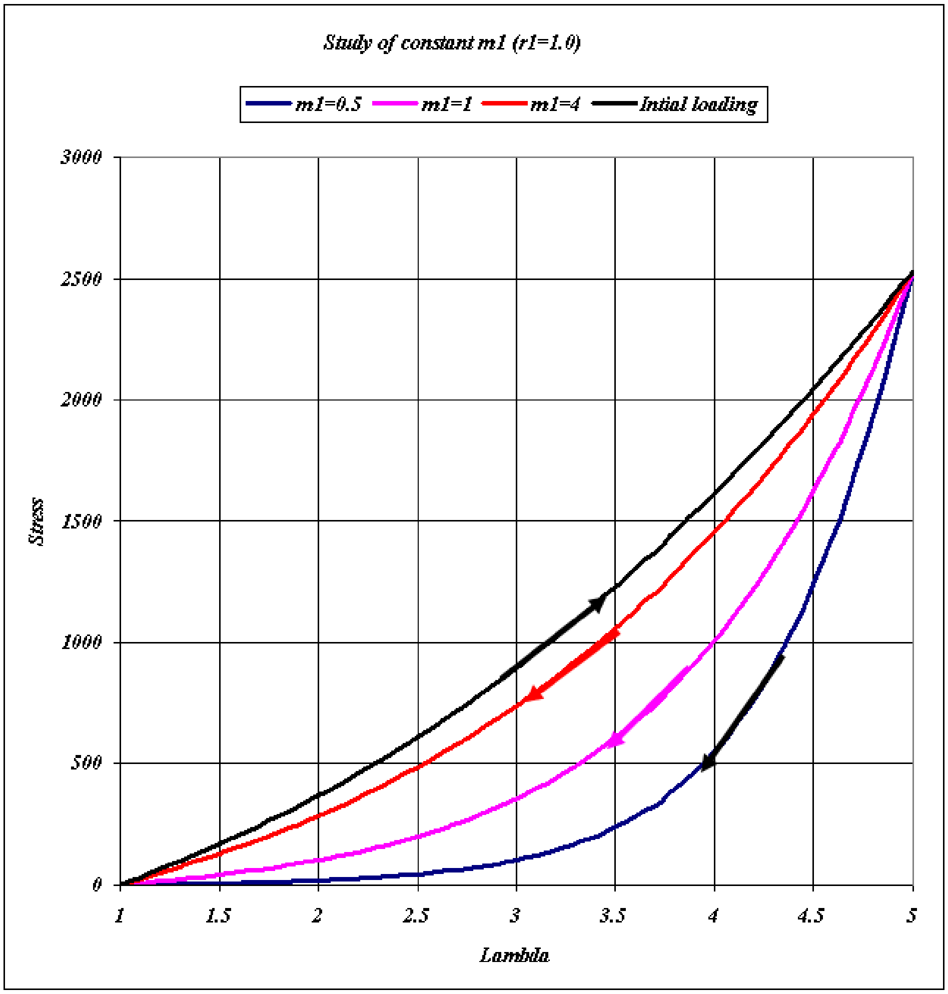

Figure 3. The effect of m_1 on unloading for uniaxial extension

The effects of r_2 and m_2 for reloading are the same as r_1 and m_1. With these four material constants the Mullins effect for most rubber-like materials can be modeled.

Determining the damage constants

The material constants can be obtained from test data and the least-square error minimization method. The test datum for i^{th} stretch ratio on unloading or reloading is $f[\lambda_1(i)]$. Hence the error between test data and calculated value is

$$t_1[\lambda_1(i)] - f[\lambda_1(i)] \quad (11)$$

For m data points, the sum of the square of errors is

$$S = \sum_{i=1}^{m} \{t_1[\lambda_1(i)] - f[\lambda_1(i)]\}^2 \quad (12)$$

By minimizing the sum of the squares of errors S, r_1, m_1, r_2 and m_2 are determined.

The initial loading curve, shown in Figure 4, is in red. The material constants are $C_1 = 75$, $\alpha = 0$. In Part One of this paper the test data are generated numerically, and $\pm 5\%$ experimental error was built into the numerical data. The raw data for unloading and reloading are shown in dots in Figure 4. The best fit that shows the Mullins effect and the raw data is shown in Figure 5. The determined Mullins damage constants are:
$r_1 = 0.8$, $m_1 = 1.0$, $r_2 = 0.5$ and $m_2 = 5$.

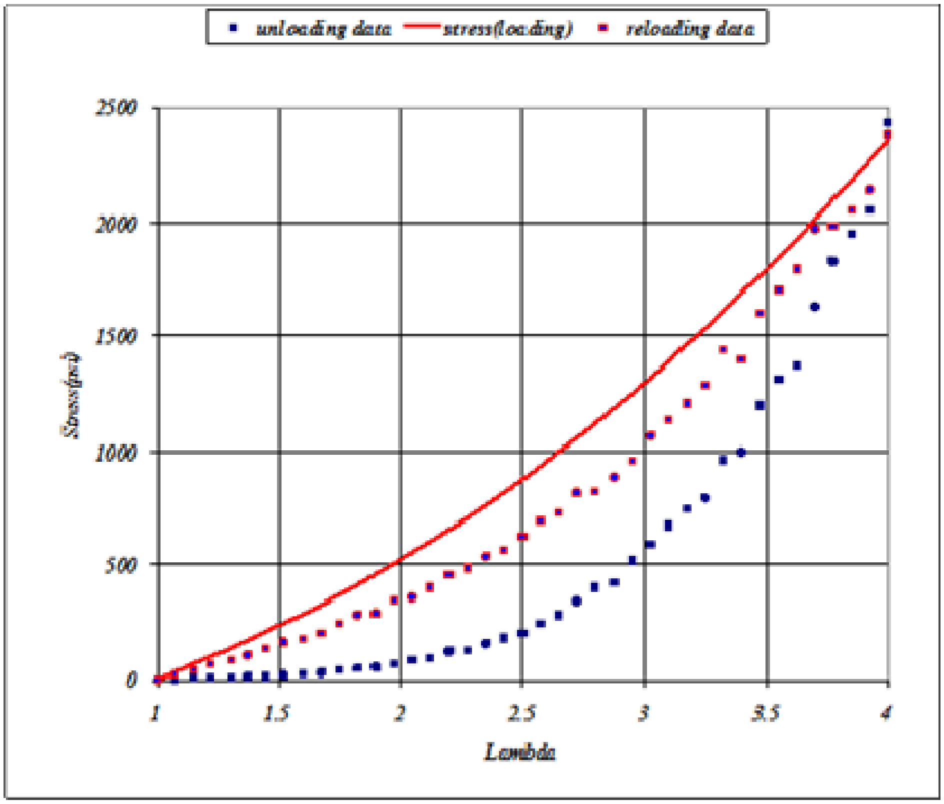

Figure 4. The stress-stretch curve for a neo-Hookean material and the numerical generated test data for unloading and reloading.

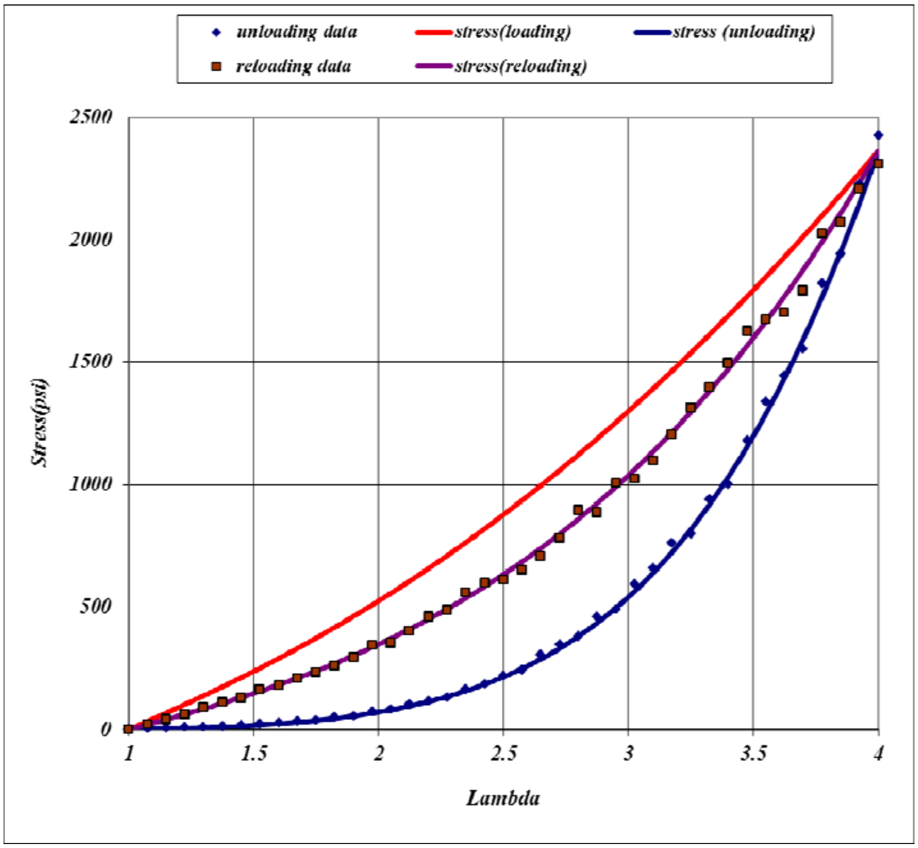

Figure 5. The best-fit unloading and reloading stress-stretch curves and the test data

LS-DYNA implementation

The formulation presented in this paper applies to one-, two- and three-dimensional problems. For general three-dimensional problems the mathematical formulation has been implemented in LS-DYNA.

The result of a cube of 0.5x0.5x0.5 subjected to uniaxial extension is obtained from LS-DYNA. The displacement at one end and the stress in the cube are shown in Figure 6. The stress-displacement plot is shown in Figure 8. The same material constants, used in the analytical calculation, are used in the LS-DYNA calculation. The displacement (Δ) can be converted to the stretch ratio (λ). $\lambda = 1 + \Delta/L$; the undeformed length of the cube is L. The results shown in Figure 1 from the analytical calculations and Figure 7 from LS-DYNA are the same.

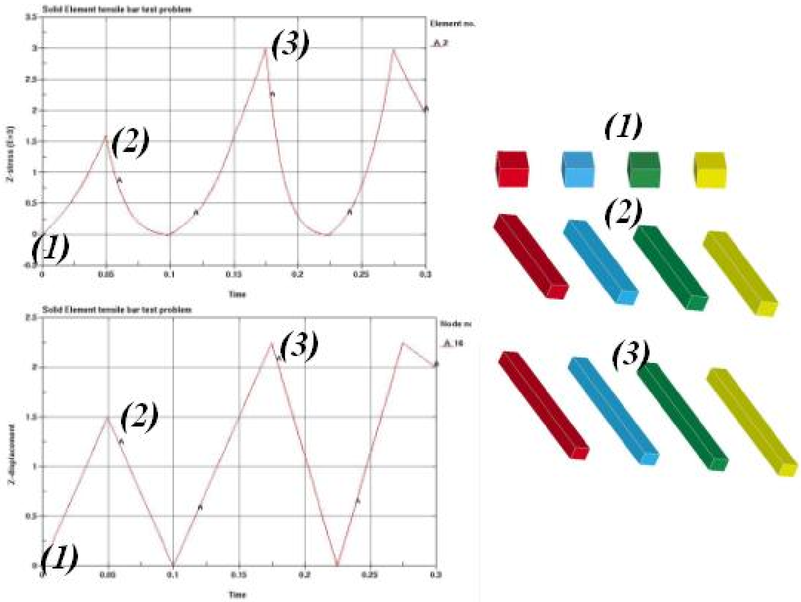

Figure 6. The results from LS-DYNA.

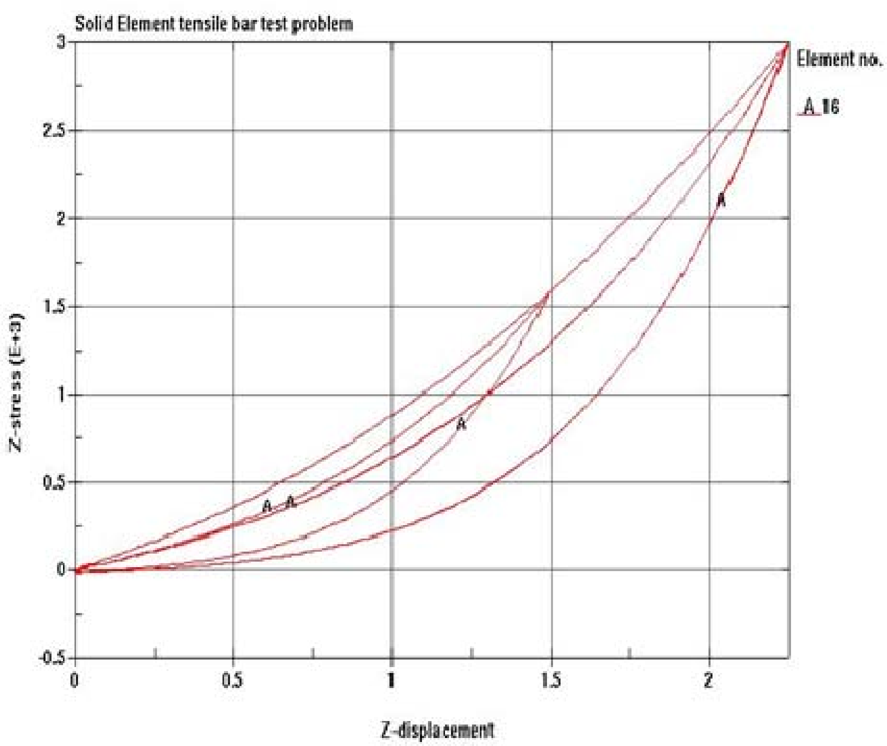

Figure 7. The stress-displacement plot from LS-DYNA.

Future work.

In the second part of this paper, the Mullins effect on biaxial loading will be studied and a biaxial test will be performed. The material constants will be determined from the biaxial test data.

The formulations and applications can be extended to various rubbers with strain-energy density represented by various constitutive equations such as: neo-Hookean, Mooney, Ogden incompressible, and Ogden compressible materials. It can also been extended to viscoelastic materials for compressible and incompressible viscoelastic materials subjected to very large deformation.

We assumed that the Mullins damage function in this paper is represented by a hyperbolic tangent function; it can be changed to other functions if needed.

References

1. W. W. Feng and J. O. Hallquist, "Numerical Modeling and Biaxial Tests for the Mullins Effect in Rubber, the 6th European LS-DYNA Conference, Gothenburg, Sweden, May 29-30, 2007.

2. W. W. Feng and J. O. Hallquist, "On Mooney-Rivlin Constants for Elastomers," the 12th International LS-DYNA Conference, Dearborn, Michigan, June 3-5, 2012.