

Advances in IGA for Sheet Metal Forming Applications

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1 Introduction

In the last years, Isogeometric Analysis (IGA) has become very popular in the scientific research community. Various types of basis functions, like Non-Uniform Rational B-Splines (NURBS), splines with T-joints (T-Splines), hierarchical B-Splines (HB-Splines), truncated hierarchical B-splines (THB-Splines), locally refined splines (LR-splines), subdivision surfaces and others are investigated by the researchers. Amongst them, the most widely used geometry representation in the Computer-Aided-Design (CAD) community is based on NURBS. Furthermore, NURBS is currently the best understood spline technology for the use of finite element analysis (FEA). It has been shown, that NURBS are particularly well suited for finite element analysis leading to qualitatively more accurate results in comparison with standard finite elements based on Lagrange polynomials.

On basis of these scientific experiences LSTC decided to work on the implementation of NURBS based finite elements in LS-DYNA a few years ago. This work will give a short overview about the general possibilities with IGA in LS-DYNA and focus on the recent advances for the analysis of Sheet Metal Forming Applications.

The paper will be organized as follows:

In section 2 the main motivation and the basic ideas of isogeometric analysis will be presented, followed by a very brief introduction into NURBS surfaces. Section 3 describes how Finite Element Analysis based on NURBS surface descriptions is realized in LS-DYNA. A benchmark example from the Numisheet 2005 conference on “*Forming of an Automotive Underbody Cross Member*” (BM2) [1], which has been analyzed by Hartmann et al. [2] in 2011 using one of the first IGA implementations in LS-DYNA, is reanalyzed in section 4 to demonstrate the progress made since then. The paper closes with a summary and an outlook about how to address further current challenges in IGA.

2 Isogeometric Analysis – some basics

This section will recall the initial motivation for the development of Isogeometric Analysis. A general definition of this term as well as some basic ideas of this method will be presented. Furthermore a short introduction about NURBS will be given.

2.1 Motivation

Setting up a model for finite element analysis (FEA) requires many steps. One of those steps is the conversion of the geometry description from computer aided design (CAD) into a suitable mesh for FEA. This is necessary as the CAD community uses geometry descriptions like e.g. NURBS whereas standard finite element analysis is generally based on low order Lagrange Polynomials for the approximation of the geometry. Therefore a re-parameterization of the initial CAD geometry is necessary, which can be quite labor cost intensive. Furthermore this meshing procedure may lead to discretization errors as the initial geometry may often not be exactly represented with Lagrange Polynomials. Isogeometric analysis aims to overcome both of these drawbacks by using directly the geometry description from CAD for the analysis. In Fig. 1 a schematic comparison of the meshing procedure between standard finite elements and isogeometric analysis is displayed. It can be seen

that the re-parameterization with Lagrange Polynomials leads to a discretization error that reduces with mesh-refinement, but will never fully vanish for this circular section.

The research on isogeometric analysis started with the focus on the question if finite element analysis could be done with NURBS, which is the most widely used geometry description used in commercial CAD packages. First promising results were presented in 2005 [3] which initiated a lot of research activity in this field thereafter. A nice introduction into the topic of isogeometric analysis can be found in the textbook by Cottrell et al. [4].

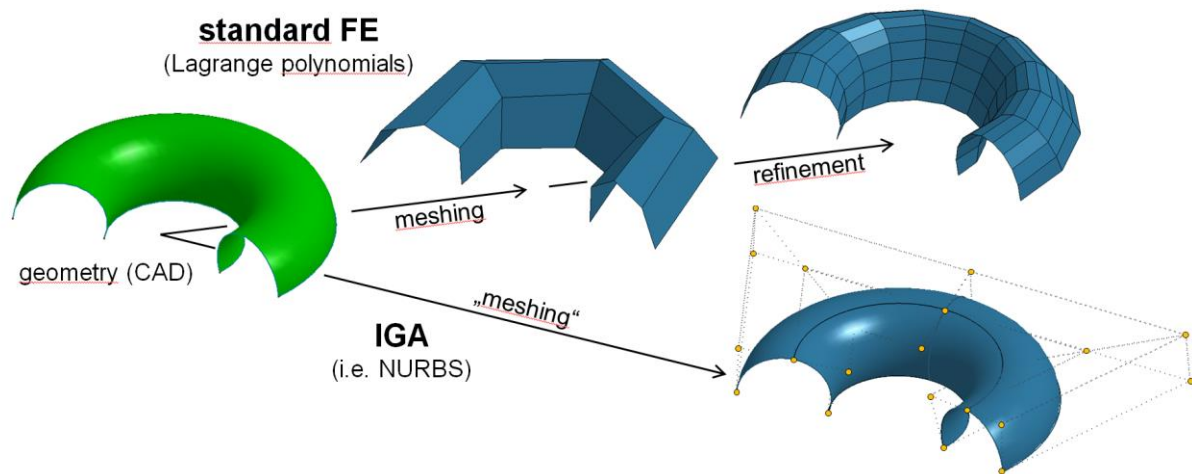


Fig.1: Comparison of meshing for standard finite elements and IGA

2.2 NURBS

To understand some significant differences of using NURBS instead of Lagrange polynomials for finite element analysis some basic properties of NURBS will be sketched in the following. For a deeper study of NURBS, the interested reader is referred to the monograph by Piegel and Tiller [5].

2.2.1 B-splines

As already obvious from its name, NURBS are built from B-splines, whose basis functions (see Fig.2) are constructed recursively until the desired degree of the functions is reached (see Equ. (1)).

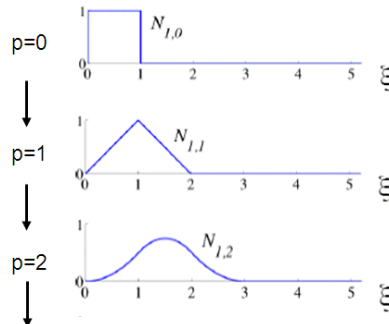


Fig.2: B-spline basis functions of order 0, 1 and 2 for uniform knot vector [4]

The recursion formula is given by

$$\begin{aligned}
 \text{for } p = 0: \quad N_{i,0}(\xi) &= \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases} \\
 \text{for } p > 0: \quad N_{i,p}(\xi) &= \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)
 \end{aligned} \tag{1}$$

where ξ_i is the i^{th} knot of the so-called “knot-vector” $\Xi = [\xi_1, \xi_2, \dots, \xi_{n+p+1}]$, which is a non-decreasing set of coordinates in the parametric space, p is the degree and n is the number of basis functions. Regardless of the degree, B-spline basis functions are always positive, they constitute the important partition of unity property and exhibit a C^{p-1} -continuity along the internal element boundaries if no multiple knot values are present in the knot-vector.

B-spline curves are created using so-called *control points* B_i , which are used as coefficients of the B-spline basis functions. It has to be noted, that the control points are normally not a part of the actual geometry which stems from the non-interpolatory nature of the B-spline basis functions. A B-spline curve $C(\xi)$ is defined through a linear combination of the B-spline basis functions with the corresponding control points.

$$C(\xi) = \sum_{i=1}^n N_{i,p}(\xi) B_i \quad (2)$$

B-spline curves may be refined (h-, p- and k-refinement) without changing the initial curve geometry.

2.2.2 NURBS surfaces

A projective transformation of a B-spline leads to a NURBS. This is achieved by introducing weights w_i at the control points. NURBS basis functions $R_i^p(\xi)$ are constructed as follows:

$$R_i^p(\xi) = \frac{N_{i,p}(\xi) w_i}{W(\xi)} \quad \text{with} \quad W(\xi) = \sum_{i=1}^n N_{i,p}(\xi) w_i \quad (3)$$

A NURBS curve is then defined in the same way as a B-spline curve, by substituting the B-spline basis functions in Equ. (2) with the NURBS basis functions.

Similarly to NURBS curves, NURBS surfaces in space can be defined. The necessary basis functions are constructed using a tensor product on the univariate basis functions (see Equ. (4)), and the final NURBS surface is then again defined through a linear combination of these basis functions with the connected control points (see Fig. 3).

$$R_{i,j}^{p,q}(\xi, \eta) = \frac{N_{i,p}(\xi) M_{j,q}(\eta) w_{i,j}}{W(\xi, \eta)} \quad \text{with} \quad W(\xi, \eta) = \sum_{i=1}^n \sum_{j=1}^m N_{i,p}(\xi) M_{j,q}(\eta) w_{i,j} \quad (4)$$

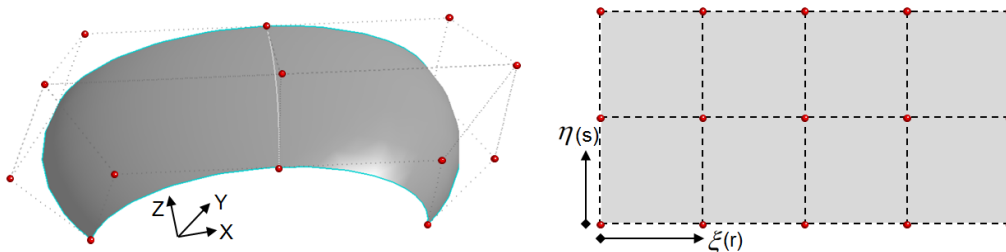


Fig.3: NURBS surface with the control points (red dots) in physical space (left) and in parametric space (right)

2.2.3 Trimmed NURBS surfaces

So far the construction of regular or standard NURBS surfaces has been discussed. But most of the time so-called *trimmed NURBS surfaces* are used in CAD packages. In addition to the definition of the NURBS surface, one or more trimming curves may be defined to trim unnecessary parts away from the initial regular patch (see Fig. 4).

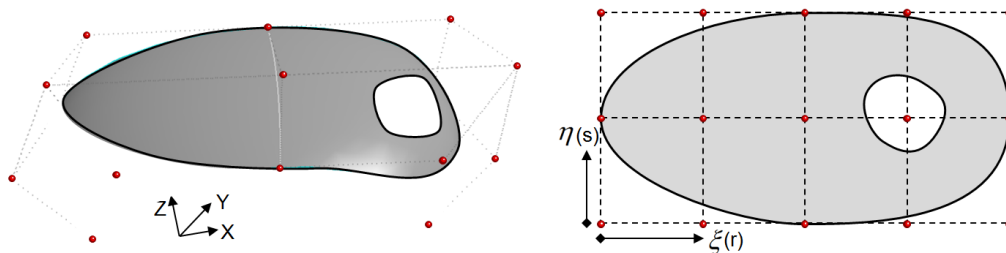


Fig.4: Trimmed NURBS surface with control points in physical space (left) and in parametric space (right)

As trimmed NURBS surfaces are standard in CAD programs, the success of IGA with NURBS is immediately related to the potential of performing finite element analysis directly on trimmed NURBS surfaces.

3 Finite Element Analysis with NURBS surfaces in LS-DYNA

In this section some fundamental features and possibilities using NURBS surfaces for FEA in LS-DYNA will be described.

3.1 A typical NURBS-patch

Doing FEA with NURBS surfaces in LS-DYNA requires the definition of appropriate NURBS patches. Therefore the keyword `*ELEMENT_SHELL_NURBS_PATCH` has to be used. Within this keyword the number of control points in the local r- and s-direction, the order of the utilized basis functions as well as the knot-vectors in both local directions need to be specified. Furthermore the grid of control points (control net) and optionally appropriate weights at the control points are defined there. These geometric informations are typically automatically generated with an appropriate preprocessor, which is currently LS-PrePost. The analyst will import a CAD geometry in form of a STEP or IGES file and then create NURBS patches for the use with LS-DYNA (see Fig 5.). These patches may then be modified, i.e. doing various refinements, without changing the initial geometry.

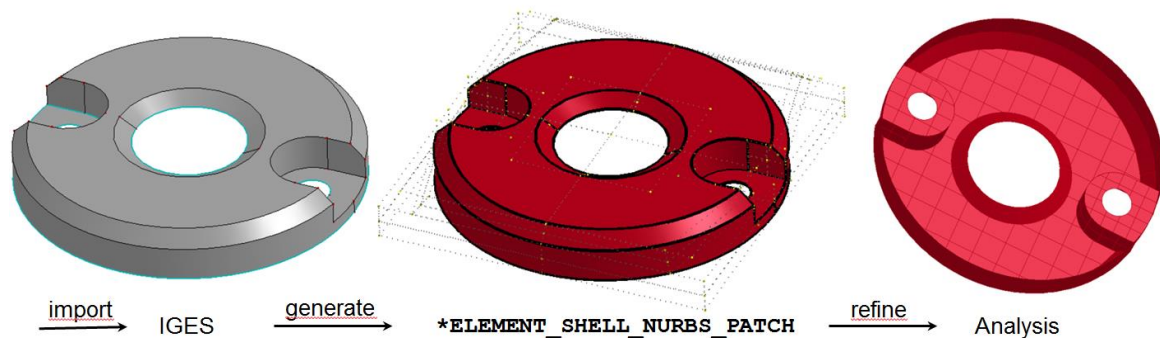


Fig.5: Workflow in LS-PrePost to generate geometry for analysis

The user can choose between different shell formulations and integration rules. Each patch will be assigned to an appropriate `*PART`, defining the material (`*MAT_XXX`) and section (`*SECTION_SHELL`) properties. Running an analysis with NURBS surfaces in LS-DYNA it is important to set `ELFORM=201` in `*SECTION_SHELL`.

3.2 Interpolation Elements

LS-DYNA automatically creates so-called *Interpolation Elements* on top of the NURBS patches. These interpolation elements are standard bi-linear shell elements whose newly created *interpolation nodes* are placed on the real surface. Currently, the interpolation elements may be used for contact treatment and post-processing. With the parameters `NISR` and `NISS` the user can define how many interpolation elements shall be created per NURBS-element. If the user specifies a negative value for `NISR` or `NISS`, LS-DYNA computes the necessary subdivisions per NURBS-element that are necessary to create an interpolation mesh with an average element edge length of $|NISR|$ or $|NISS|$ (see Fig. 6).

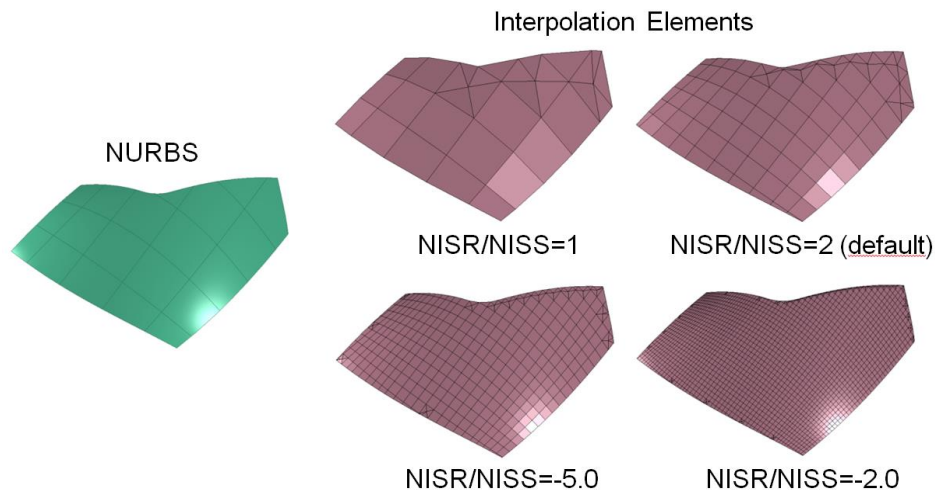


Fig.6: Bi-quadratic NURBS patch and interpolation elements dependent on the parameter NISR and NISS

It is important to notice, that the constructed interpolation nodes are dependent nodes with respect to the control points such that their motions are fully constrained to the underlying NURBS patch. When using the interpolation elements for contact, the contact forces are first evaluated at the interpolation nodes, but then transferred to the primary degrees of freedom (DOF) at the control points. The actual analysis is exclusively performed using the NURBS elements and their corresponding DOFs at the control points. For post-processing, the information at the integration points of the NURBS elements are mapped onto the interpolation mesh, such that the standard post-processing can be used.

3.3 Analysis capabilities

3.3.1 Shell formulations

Basically three different shell formulations are available for using NURBS shell elements, which can be defined via the parameter FORM in the `*ELEMENT_SHELL_NURBS_PATCH` keyword card. The first one is based on the Reissner-Mindlin shell theory and uses three translational and three rotational degrees of freedom, similar to the classical shell elements existing in LS-DYNA. As second choice a rotation free shell formulation based on the Kirchhof-Love theory is available. This makes use of the higher continuity of the NURBS basis functions along the element boundaries and allows a saving of 50% of the degrees of freedom, which makes this shell formulation in particular attractive for implicit analysis. The third shell formulation is a mixture of both, leading to a hybrid formulation. It permits to individually define control points with or without rotational degrees of freedom. As the continuity of the shape functions in a NURBS patch drops to C^0 at the patch boundaries, the hybrid shell formulation provides the possibility to couple regular NURBS patches along their patch boundaries by locally introducing rotational degrees of freedom while still exploiting the higher continuity in the interior of the patch.

3.3.2 Integration rules

Currently three different integration rules are available, which are invoked using the parameter INT. A uniformly reduced Gauss integration as the default setting and a full Gauss integration scheme are available for all NURBS elements, irrespective of the order of the shape functions. For C^1 -continuous bi-quadratic NURBS an additional, reduced patch-wise integration rule based on the work of Adam et al. [6] is available. This is characterized by the least amount of necessary integration points and better addresses the appearance of some shear locking tendencies of quadratic NURBS.

3.3.3 Trimmed NURBS

Following the work of Nagy and Benson [7], the support of performing the analysis directly on trimmed NURBS surfaces has been added to LS-DYNA. This unalterable necessity for a general use of IGA in industrial applications is available since version R9. An unlimited number of trimming loops may be added to the general patch description to define the outer boundary of the desired surface and to cut out various holes in the interior. In Fig. 6 a plate with a circular hole has been analyzed to compare the performance of the trimmed NURBS implementation with respect to the standard well-established bi-

linear shell elements. It can be seen, that both simulations basically give very similar results and given the fact that using trimmed NURBS allows the use of a regular spaced grid of control points, the displacement field can be represented a little smoother than with standard FEA.

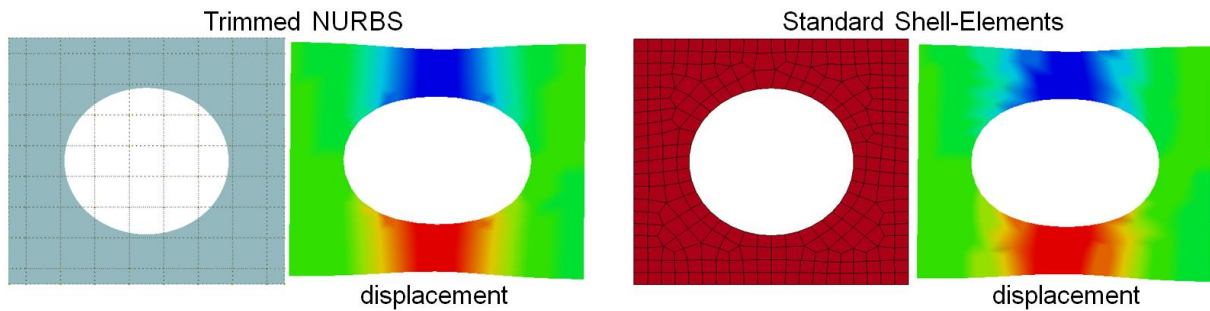


Fig.7: Plate with hole: Comparison of trimmed NURBS (left) vs. standard Shell-Elements (right)

3.3.4 Miscellaneous

Besides the already mentioned possibilities, the NURBS-based finite elements can be used in explicit as well as in implicit analysis. They are available in SMP (shared memory parallel) and MPP (massively parallel processing), a NURBS contact implementation is available by setting IGACTC=1 in `*CONTROL_CONTACT`, boundary conditions may be applied directly to the control points or to any location on the surface by constraining a node via `*CONSTRAINED_NODE_TO_NURBS_PATCH` onto a NURBS patch. Many material models from the LS-DYNA material library are available for IGA. Furthermore, mass scaling [8] has been implemented for the NURBS elements and they can be used as rigid bodies.

4 Sheet metal forming with IGA

In this part the current possibilities of LS-DYNA in view of sheet metal forming applications will be presented. The Numisheet 2005 Benchmark on “Forming of an Automotive Underbody Cross Member” (BM2) [1], which was already analyzed by Hartmann et al. [2] in 2011 with one of the very first versions of IGA in LS-DYNA is reviewed and the today's numerical performance is compared with respect to the first attempts.

4.1 Example: Underbody Cross Member – then and now

4.1.1 Example setup

The setup of the forming process is shown in Fig. 8. Only the deep drawing process is simulated, where the tools (upper die, binder and lower punch) are modelled with rigid elements. For all analyzed models, only the discretization of the blank was varied, while all remaining settings remained unaffected. It was not intended to actually optimize the result with respect to the original Numisheet benchmark but to compare the results between standard FE and NURBS elements in LS-DYNA using identical boundary conditions and properties.

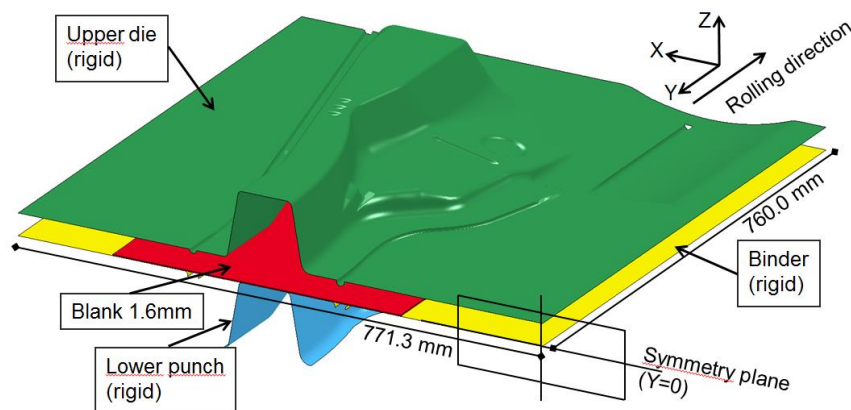


Fig.8: Numisheet 2005 – BM2 setup [2]

4.1.2 Results

The study by Hartmann et al. [2] has shown, that for qualitatively good results, an average mesh size of 2mm for the blank is needed when using standard, fully integrated bi-linear shell elements (ELFORM=16) and an average mesh size of 4mm when using bi-quadratic NURBS elements. Therefore the example has been re-computed with these two element formulations and their necessary level of discretization. In Fig. 9 the equivalent plastic strain distribution at the end of the forming step is shown. It can be seen that both element formulations lead to very similar results.

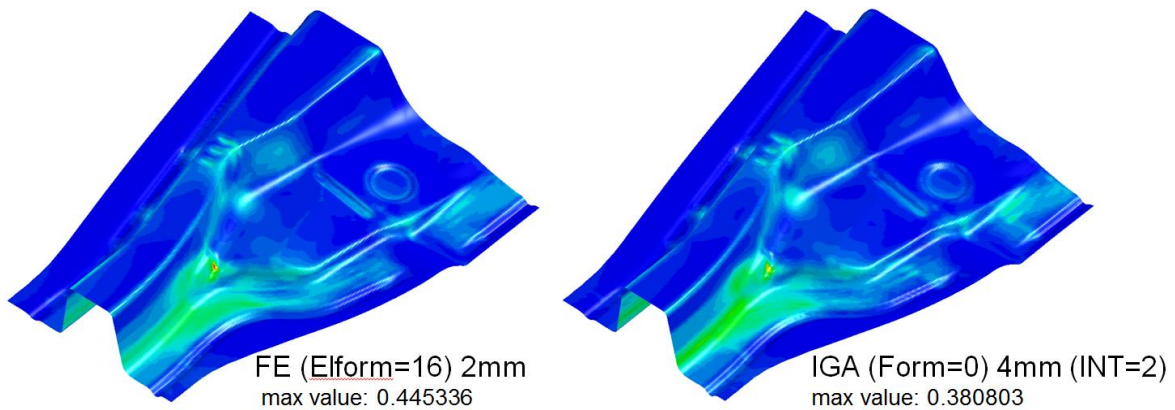


Fig.9: Comparison of equivalent plastic strain – standard FE (left) and NURBS (right)

Furthermore the evaluation of the thickness change is displayed in Fig. 10, again showing very similar results.

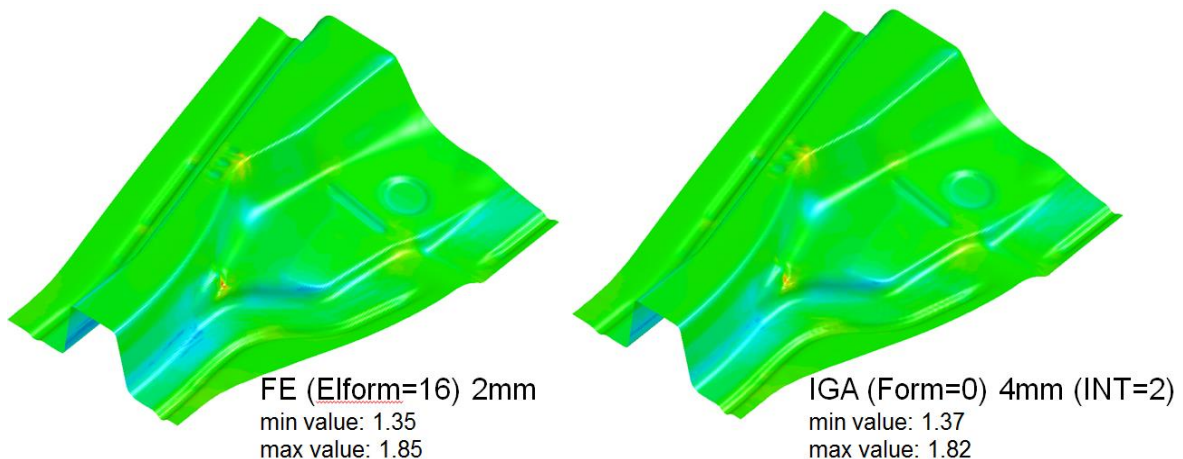


Fig.10: Comparison of thickness change – standard FE (left) and NURBS (right)

4.1.3 Numerical performance

In the study from 2011 [2], the analysis could only be done using an SMP, double precision version of LS-DYNA, without having the possibility of computing the thickness change nor using mass scaling.

2011	smp (4 AMD 2.2 GHz) no mass scaling no thickness update		2016	mpp (4 Intel 2.67 GHz) mass scaling (app. 10%) with thickness update
CPU-time (h:min)			CPU-time (h:min)	
- Std-2mm:	21:06	17.4 %	- Std-2mm:	3:41
- P2-4mm:	14:31	15.1 %	- P2-4mm_i0:	2:12 (59.7 %)
			- P2-4mm_i2:	1:34 (42.5 %)

Fig.11: Comparison of computing time: 2011 [2] vs. 2016

The comparison of the computing time is shown in Fig. 11. It can be seen, that the computing time going from SMP (double precision, ncpu=4) to MPP (double precision, np=4), using approximately 10% mass scaling and activating thickness update reduces to 17.4% for the standard elements and to 15.1% for the NURBS elements. A further reduction of the computing time without any loss of accuracy can be achieved by switching from uniformly reduced integration (P2-4mm_i0, INT=0) to the patch-wise reduced integration scheme (P2-4mm_i2, INT=2). Again it can be verified for this example, that the analysis with bi-quadratic NURBS elements with an average mesh size of 4mm needs less than 60% of the computing time needed to perform the calculation with standard bi-linear shell elements with an average mesh size of 2mm. It has to be noted that these mesh sizes are necessary to achieve good and comparable results. Although the computational cost per NURBS element is significant higher than per standard element, the larger mesh size, leading to less elements and a larger timestep size can easily compensate this extra effort.

4.1.4 Speedup

The example with bi-quadratic, 4mm NURBS elements and reduced patch wise integration rule (P2-4mm_i2) has been used to analyze the speedup of the MPP implementation of IGA in LS-DYNA. The model was computed on an Intel Xeon 2.80 GHz cluster using 1, 2, 5, 10, 20 and 40 compute nodes.

# of procs	Time [s]	Time [h:min:s]	Speedup S_p
1	8147	2:15:47	1.00
2	4582	1:16:22	1.78
5	2671	0:44:31	3.05
10	1855	0:30:55	4.39
20	1046	0:17:26	7.79
40	643	0:10:43	12.67

$$S_p = \frac{T_1}{T_p} \quad \text{with:}$$

T_1 : Time on 1 proc

T_p : Time on p procs

Table 1: Computational time using different number of procs

It is clear that there can never be an ideal speedup, as there are always fractions that cannot be parallelized. Therefore the real speedup is defined

$$S_p = \frac{T_1}{T_1 \cdot \left((1-f) + \frac{f}{p} \right)} \quad (5)$$

where T_1 is the time on 1 proc, f is the fraction of T_1 that can be parallelized and p is the number of procs. In Fig. 12, the realized speedup for this example is plotted against the ideal speedup and two real speedup curves using Equ. (5) with a parallelization fraction f of 90% and 95%. It can be seen that the IGA implementation in LS-DYNA performs quite well.

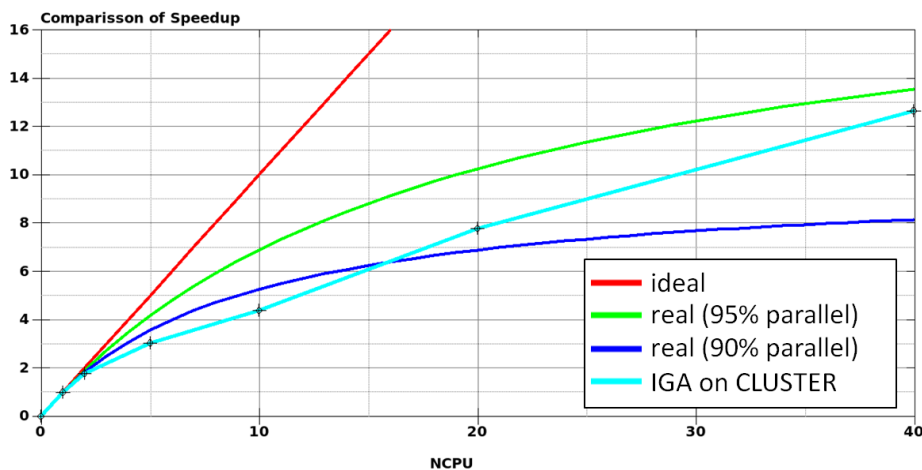


Fig.12: Speedup of IGA on the cluster compared to ideal and real speedup

4.2 Supported Keywords

Recently the support of some important keywords that are frequently used in the context of sheet metal forming has been added to the IGA capabilities. They will be listed below:

- ***PART_MOVE**
- ***CONTROL_FORMING_AUTOPOSITION_PARAMETER**
- ***CONTROL_ADAPTIVE**
(adaptivity for standard shells is supported when NURBS elements are in the model, adaptivity for NURBS is not yet available)
- activation of frequently used anisotropic material models like ***MAT_036** and ***MAT_133**

4.3 Discussion

The IGA implementation in LS-DYNA allows the deep drawing of sheet metals giving comparable results to the ones achieved with standard finite elements. A reasonable well scaling of the MPP implementation has been shown. However, there are still various features missing for IGA in LS-DYNA to be productively used in industrial applications, like:

- writing *dynain-file* for the simulation of several processing steps
- adaptivity for the NURBS elements
- trimming of the blank after forming

So the start is made and feedback like bug reports and requests are welcomed and necessary to further improve the capabilities of IGA in LS-DYNA for this specific application.

5 Summary

The general idea of Isogeometric Analysis (IGA) and the current possibilities of this method in LS-DYNA have been presented. By means of a Numisheet benchmark example the current performance of the IGA implementation with respect to deep drawing was compared to the one achieved in the study by Hartmann et al. [2]. The comparison of the results between the well-established fully integrated bi-linear standard shell elements (ELFORM=16) and bi-quadratic NURBS elements has shown, that the same quality of results can be achieved with larger mesh sizes for the NURBS elements. Hence a larger stable timestep size can be used, which, together with a smaller number of necessary elements, finally leads to a reduction of the computational time. Furthermore the scalability of the MPP implementation has been tested on a cluster with up to 40 compute nodes, which showed a reasonable well performance. Recently the support for other keywords, frequently used in the context of sheet metal forming simulation was added to the IGA capabilities in LS-DYNA.

Although the results achieved in this paper are very promising, there is still a lot of work left to be done to make the IGA technology in LS-DYNA available for general use in industrial applications. The most important topics that need to be addressed are the support of ***INTERFACE_SPRINGBACK** and the trimming of the stamped part.

6 Literature

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