Numerical modelling of symmetric and asymmetric punching and post-punching shear responses of RC flat slabs

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1 Introduction

Structural robustness has become an important requirement in the design of civil engineering structures. It has been defined as the insensitivity of a structure to initial local failure [1]. Response of different structural forms differ due to the various possible mechanisms developed after an initial failure of a structural component. There exists a gap in knowledge of the response of reinforced concrete (RC) flat slab structures after an initial local failure. Flat slabs are RC slabs supported directly on columns without beams (fig. 1a), unlike conventional RC framed structures. Bending moment and shear transfer from slabs to columns poses challenges in flat slab structures due to the possible occurrence of punching shear failure at very small rotations. The provision of punching shear reinforcement increases punching shear strength and deformation capacity. However, errors in design, abnormal loading of the structure, malevolent or accidental actions may lead to loss of a column or the failure of a connection. Loss of a column or failure of a connection could then lead to a partial collapse (figure 1b) or a complete collapse of the structure. The primary mechanisms initiated after such local failure would include post-punching shear, tensile membrane action and asymmetric punching of neighboring connections. Inertia and material strain rate effects are also very influential on the response of the flat slab structural system at this phase.

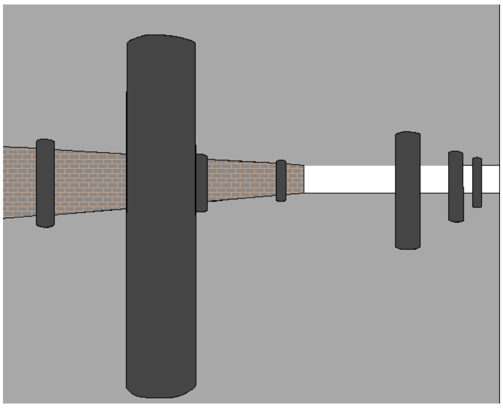


Fig.1: (a) Typical flat slab structure *(b) Progressive collapse of Pipers Row Car Park [2]*

Punching shear and post punching responses of flat slab connections are dynamic and nonlinear in nature, though they have been experimentally and analytically modelled as static nonlinear problems using displacement based models. While displacements and rotations resulting from punching shear are small, the post punching mechanism involves large displacements. Significantly high material and geometric nonlinearities develop during the post punching shear phase of failure making it difficult to model the response of these connections numerically using implicit-static finite element approaches. This is primarily due to numerical convergence problems which would result during iterative steps required to establish equilibrium. However, explicit finite element analysis does not require the establishment of equilibrium. Nodal accelerations and displacements are solved for directly using time steps which are of many orders of magnitude smaller than those adopted for the implicit analysis.

Previous researches involving the numerical modelling of progressive collapse in flat slab structures have seen the use of connector elements (discrete beam elements, connector beams, springs or connector spheres) for the simulations of the punching and post punching shear mechanisms [3,4]. Load-deformation and load-failure relationships adopted for these connectors are obtained from analytical models. Application of these models tend to neglect or inadequately predict the contribution of various mechanisms to the overall capacity of connections when applied to system models. The interaction between these mechanisms also contribute significantly to the overall capacity of the connection during failure. Explicit modelling of connections using solid and beam elements allow for an adequate consideration of the various mechanisms as well as their interaction.

Research currently being carried out at the University of Surrey, aims at developing novel analytical and numerical methods towards assessing structural robustness of RC flat slab structures, with a view to improving the safety of structures incorporating this structural system. Presented in this paper, is the numerical procedures adopted for modeling of punching shear and post punching shear response of RC flat slab connections using the finite element (FE) software LS-DYNA. Numerical procedure adopted is validated using isolated RC flat slab test specimens available in literature.

LS-DYNA was adopted for this study because of its;

- explicit time integration methodology which is ideal for quasi-static and dynamic analyses of systems with significantly high material and geometric nonlinearities,
- rich variety of element formulations and material models,
- and erosion capabilities of its built-in or user defined material models, allowing for an effective simulation of the disconnection of certain structural components from others when the prescribed criteria are met.

2 Methods

2.1 Element formulations

Modelling of column-slab connections was carried out using solid (three dimensional) elements for concrete and beam (one dimensional) elements for steel reinforcement. The 8 noded hexahedron element with its default constant stress solid element formulation was adopted for the solid elements due to its simplicity and versatility. For the one dimensional beam elements the Hughes-Liu beam element formulation was adopted. The Hughes-Liu beam element formulation was chosen since it incorporates finite transverse shear strains in addition to its bending and membrane capability.

2.2 Constitutive material models

Constitutive material models available for use in LS-DYNA for modelling concrete with solid elements include material models 5, 14, 16, 25, 72, 72R3, 96, 84, 145 and 159. Of these material models, 72R3, 84 and 159 are the most commonly used because they allow for easy input material parameters. Performances of these three material models have been well compared in studies by [5,6]. For the purpose of this study, the material model 84 was adopted over material models 72R3 and 159 because it allows for the direct input of major material properties. The material type 84 (*MAT_WINFRITH_CONCRETE) was developed by Broadhouse and Neilson (1987) [7]. It was incorporated into LS-DYNA in 1991. It is a smeared crack model based on the Ottosen plasticity model [8]

The Winfrith concrete material model caters for both triaxial compression and triaxial extension using the third stress invariant [7]. It includes concrete tensile cracking in three orthogonal planes for each element and strain softening in tension. Input parameters for normal strength and siliceous aggregate concrete in the Winfrith concrete model are relatively straight forward. Input parameters for tangential modulus (T.M) and uniaxial tensile strength (f'_{ct}) of concrete were obtained using equations 1 and 2. Both relationships are as used in psi by [5] but presented here in MPa. Strain rate effects could be turned on or off for either a quasi-static or dynamic analysis respectively. For the quasi-static analysis, "FE" material input parameter represents the crack width (strain) at which crack-normal tensile stress goes to zero. A constant value of 1.27×10^{-4} was adopted as recommended for normal strength and siliceous aggregate concrete [9].

$$T.M = 4733\sqrt{f'_c} \qquad (MPa) \tag{1}$$

$$f'_{ct} = 0.581\sqrt{f'_c} \qquad (MPa) \tag{2}$$

The Winfrith concrete model has no in built element erosion capability. Erosion for this material model was hence incorporated using the *MAT_ADD_EROSION keyword of LS-DYNA. "Maximum principal strain at failure" and "shear strain at failure" were the only two erosion criteria adopted in models where erosion was incorporated. This was due to the assumption of only flexural and shear failure modes at the connection.

*MAT_PLASTIC_KINEMATIC (material model 3) was used in modelling of the material properties of the steel reinforcements. This material model is one usually adopted for modelling isotropic and (or) kinematic hardening plasticity. It is compatible with the Hughes-Liu beam element and strain rate effects could be incorporated into this model. Erosion was incorporated directly into this material model through plastic strains.

2.3 Arbitrary Lagrangian-Eulerian Coupling

*CONSTRAINED_LAGRANGE_IN_SOLID keyword was used to model interaction between solid and beam element. It is based on an arbitrary Lagrangian-Eulerian (ALE) constitutive model. With this approach to coupling there is no need for the tedious task of sharing solid and beam element nodes or incorporating one dimension discrete elements into the model. The distribution of coupling points along the surface of each Lagrangian element (beam element) was done based on the relative mesh sizes of the Lagrangian element and the ALE elements (solid elements). A minimum of two coupling points in each ALE element was allowed for. The coupling type 2 (CTYPE=2) was adopted. Coupling was done in both compression and tension.

2.4 Test specimens

Isolated flat slab test specimens PM2, PM4, PM12 [10] and SS [11] were obtained from literature where their static flexural, punching shear and post punching shear responses were determined under symmetric support and loading conditions. Specimens PM2 and PM4 were without integrity reinforcement and differed primarily in the percentage of flexural reinforcement they contained. PM12 and SS both contained four bars of integrity reinforcement passing through the column centre. Slab specimens characteristics and properties are as shown in figure 2 and table 1.

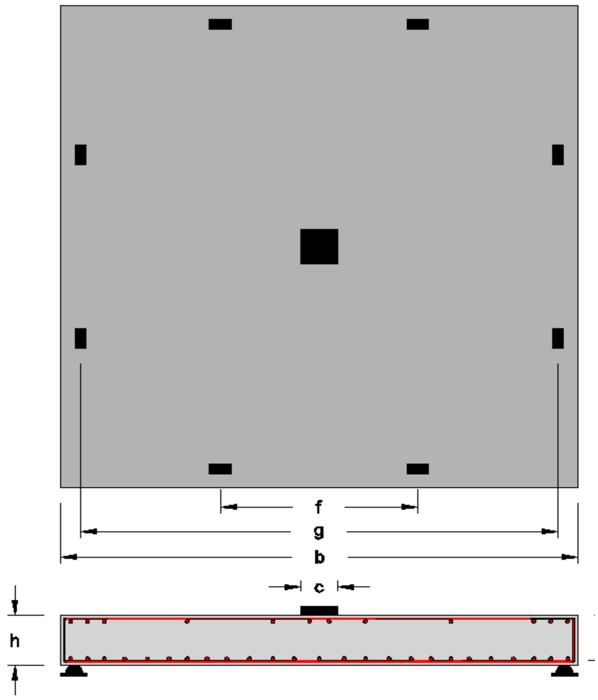


Fig.2: *Typical slab specimens characteristics*

Parameter	PM2	PM4	PM12	SS
b (m)	1.5	1.5	1.5	2.3
g (m)	1.38	1.38	1.38	2.0
f (m)	0.575	0.575	0.575	0.75
c (m)	0.13	0.13	0.13	0.225
h (m)	0.125	0.125	0.125	0.2
d (m)	0.102	0.102	0.102	0.16
f_c (MPa)	36.5	36.8	32.4	26
f_{ct} (MPa)	2.8	3.0	2.6	3.35 *
ρ (%)	0.49	1.41	0.82	**
f_{sy} (MPa)	601	601	601	420
f_{su} (MPa)	664	664	664	723
$f_{sy.int.}$ (MPa)	-	-	527	420
$f_{su.int.}$ (MPa)	-	-	629	723
$\Phi_{int.}$ (m)	-	-	0.014	0.016

* Splitting tensile strength
** 16mm Φ bars provided at a spacing of 160mm within 0.64m from the column centre and at 200mm beyond

Table 1: *Slab specimens properties*

2.5 Symmetric slab specimens modelling, loading and analysis

Due to symmetry of test specimens along the two orthogonal planes, quarter three dimensional FE models were developed for each test specimen as shown in figure 3a. Flexural, integrity and perimeter reinforcements were all modelled explicitly using different parts represented by colors blue, yellow and green respectively (figure 3b). Loading of specimens were carried out by gradually increasing slab displacement at loading points, as applied in tests. *BOUNDARY_PRESCRIBED_MOTION_SET LSDYNA keyword was adopted for this purpose. This keyword allowed for the definition of nodal motion imposed on sets of nodes. A gradual constant displacement of 0.0006 metres for every time step (time step interval of 1 second) was allowed for. This was to ensure that dynamic effects developed during analyses were negligible.

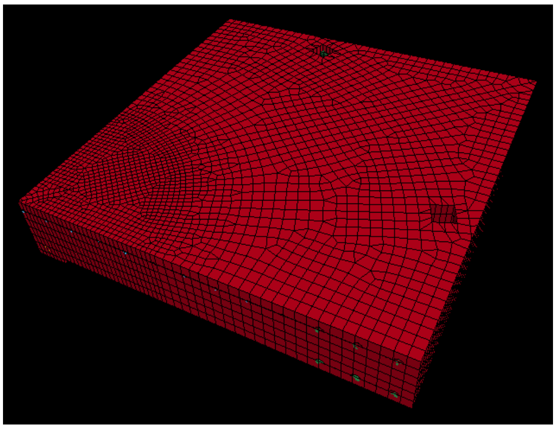
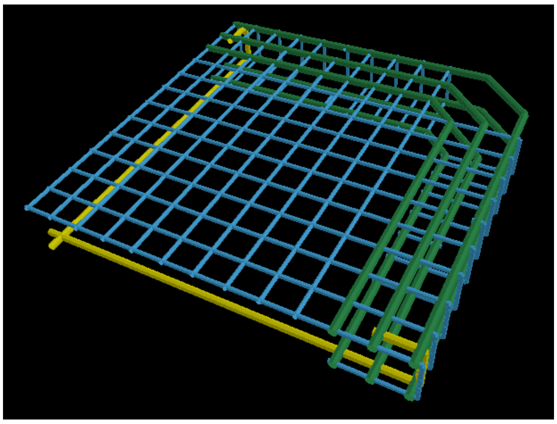


Fig.3: (a) Finite element meshing for slab specimen PM12



(b) Explicit modelling of reinforcements for slab specimen PM12

Quarter FE models were analysed numerically using a quasi-static displacement controlled approach and their flexural, punching shear and post punching shear responses obtained. Sensitivity analyses were carried out to obtain the optimum element characteristics for punching shear capacity, residual capacity just after punching and initial post punching capacity. Due to limited analysis time a complete assessment of influence of the various parameters on peak post punching capacity could not be carried out. Element lengths of 10 and 20mm, element depth to slab depth ratios (e.d/s.d) of 0.25, 0.1667 and 0.125 were used for the parametric assessment. Inputs of 0.02 and 0.1 were also adopted as inputs for both erosion failure conditions for Winfrith concrete.

2.6 Asymmetric slab specimens modelling and analysis

Most assessments on robustness have been based on scenario independent approaches such as the sudden removal of a column. A linear FE assessment carried out by [12] on a symmetrically supported flat slab showed a 25% to 35% increase in the load carried by adjacent columns after the static removal of an internal column. Increase in loading and a reduced connection strength due to eccentricity increases the possibilities of adjacent connections punching asymmetrically. The methodology here adopted for the numerical modelling of asymmetric punching and post punching shear response of flat slab column connections were quite similar to those of symmetric connections. However, due to varying slab displacements at equal distances from the four column faces, displacement ratios were adopted for asymmetric cases. Possible estimates of displacement ratios were assessed using a continuous flat slab system model as shown in figure 4. Continuous flat systems were developed based on the characteristics of isolated slab specimens adopted. The gradual loss of opposite connections after the loss of an internal column was simulated statically through a nonlinear static FE analysis.

Slab displacements at the side of the lost column (critical side) were assigned a displacement factor of one. Slab kinematics were assessed at various steps of analysis. Slab displacement contours after the complete loss of the connections are as shown in figure 4. Slab displacement curve showing displacements from the mid-spans on either side of column 2 is as present in figure 5. Figure 5 shows displacement curves at the point of initiation of punching shear failure and at complete loss of the connection. A displacement of 0.116m was observed at column 2 after the complete loss of the connection. It should be noted that this assessment was carried out without consideration of post-punching resistance of the failing connections as well as the failure of other connections.

Ratios of displacement at distances from the column centre at the opposite and adjacent sides to those at equal distances of the critical side were obtained. Curves for these displacement ratios after complete connection failure are as shown in figure 6. Displacement ratios decreased with increase in distance from the column face. Figure 7 shows the development of the displacement ratio through the various steps of analysis at three points 1.12d, 3.04d and 6.72d from the column face. The negative factors observed in figure 7 resulted due to positive displacements (uplifts) at the opposite side prior to the failure of the connection and at early steps of the analysis. Results of presented in figures 4 to 7 were based on flat slab system developed from isolated slab specimen PM4. Based on these observations, average displacement ratios of 0.65 and 0.75 were adopted for the opposite and adjacent sides respectively. These ratios were applied in the numerical assessment of asymmetric punching in isolated flat slab models.

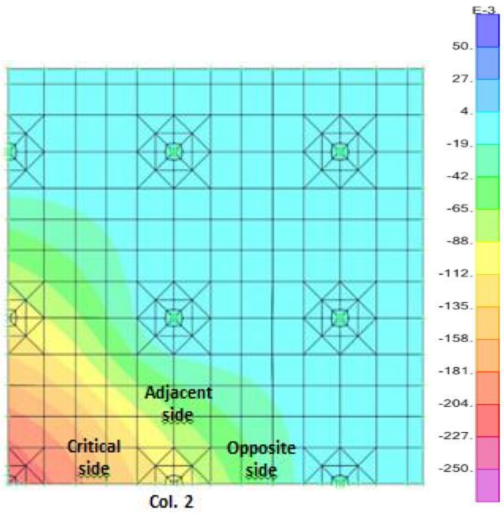


Fig.4: Slab displacement contour after complete failure of adjacent connections (displacement in metres)

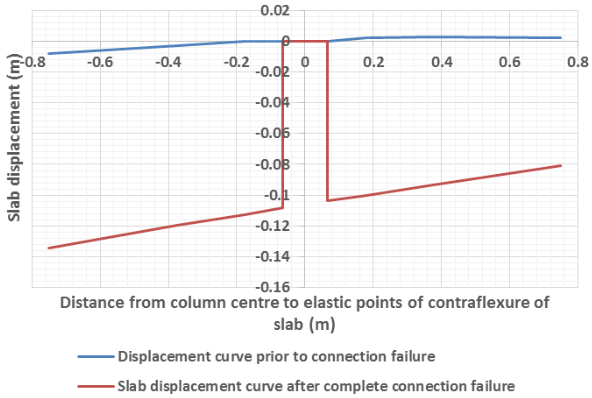


Fig.5: Slab displacement around column

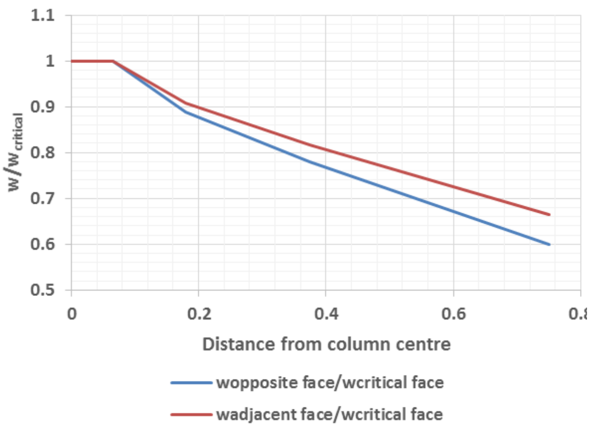


Fig.6: Displacement ratios for opposite and adjacent faces after complete failure of adjacent connections

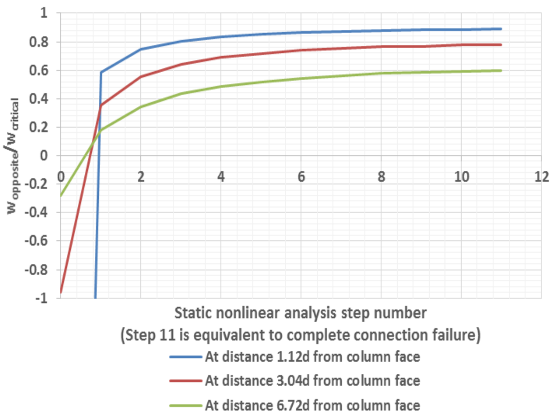


Fig.7: Displacement ratios for opposite face at various analysis steps

3 Results

3.1 Sensitivity study

Results of sensitivity studies gave adequate insight into how model characteristics influenced model behaviour in punching shear, residual punching shear and post punching shear. Inclusion of very small values of erosion to concrete material model by means of failure strain criteria gave strength estimated much lower than the actual when these criteria were met. As shown in figure 8 for the PM12 slab specimen, inclusion of the erosion criteria influences all three connection properties. Hence, when there is need to model disconnection of members due to local damage without influencing strength properties, much higher values of failure criteria should be adopted though these strain values may seem mechanically unrealistic. This sensitivity study was carried out using element length of 0.01m and e.d/s.d ratio of 0.167. Though it is expected that smaller values of element length and e.d/s.d ratios may lead to better prediction of strength at smaller values of strain failure criteria, these would significantly increase computation time and cost, especially when applied to system models.

Sensitivity studies also showed that element depth to slab depth ratios of 0.1667 gave the best prediction of punching shear strength with a percentage difference of 0.03% (figure 9). Higher e.d/s.d led to punching shear strengths higher than actual and the opposite case was observed for lower values of e.d/s.d. Variations in e.d/s.d had no influence on the residual punching shear and post-punching shear responses.

Element length of 0.010m gave the best prediction of punching shear strength as shown in figure 10. Smaller element sizes gave lower punching shear strength predictions. There was no observed influence of variations in element sizes on the residual punching shear and post punching shear responses. Element sizes lower than 0.010m such as a 0.005m element size was considered to fine to be incorporated into this sensitivity assessment

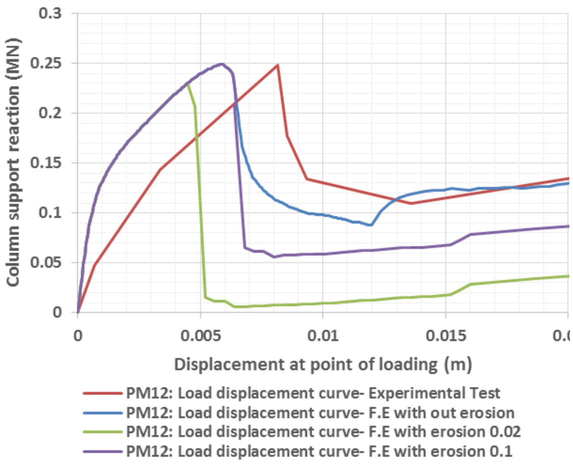


Fig.8: Comparison of model responses with and without erosion

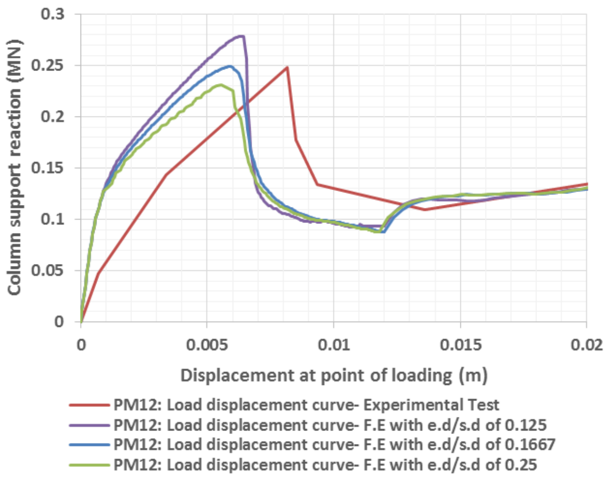


Fig.9: Comparison of model responses with varying element depth to slab depth ratios

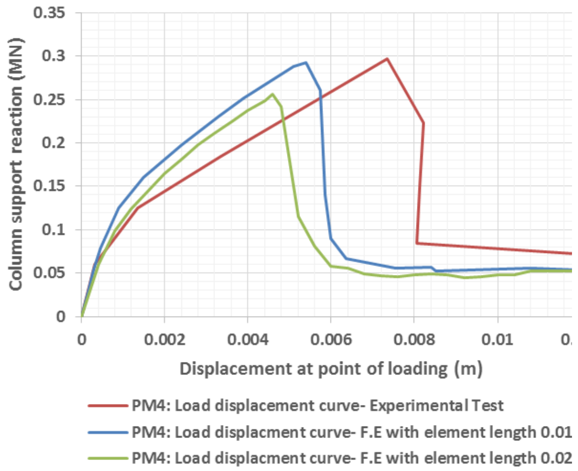


Fig.10: Comparison of model responses with varying element length

3.2 Symmetric assessment

Sensitivity studies showed 0.01 element length, 0.1667 e.d/s.d and a zero erosion criteria to be the combination with the best prediction of punching shear, residual punching shear and post punching responses. This model characteristics were used for a complete analysis of the PM2, PM4, PM12 and SS specimens. Results obtained are as shown in figure 11 to figure 14. Punching shear capacity for the various specimens gave percentage differences between 0.03 and 2 percent. Residual punching shear strengths gave 0.03, 21, 19 and 1.1 percent differences for PM2, PM4 PM12and SS respectively and peak post punching shear strength gave percentage differences between 0.03 and 3 percent. No fracture of flexural or integrity reinforcements was observed in the F.E analysis which was also the case in tests. Analysis of SS model terminated abruptly at a displacement of 0.1526m due to nodes with “out of range forces”.

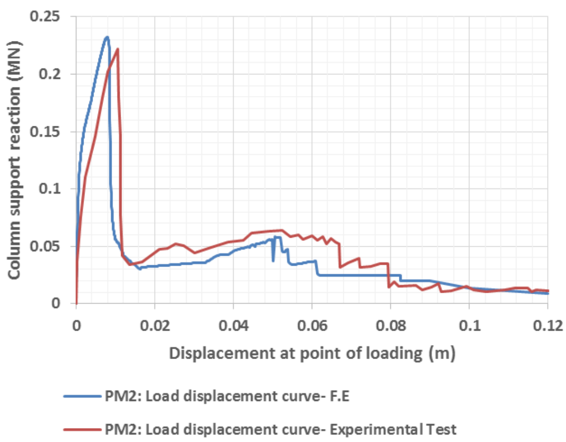


Fig.11: Symmetric PM2: Load displacement curve

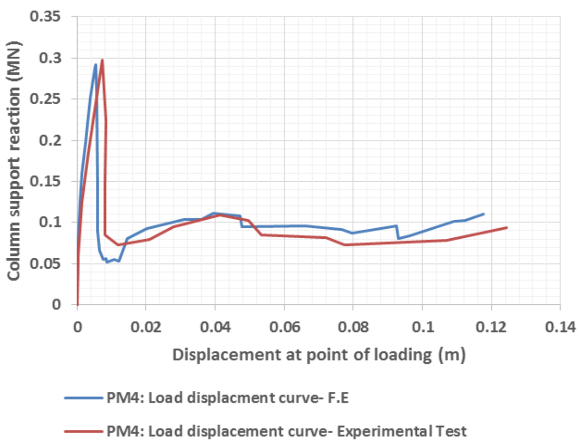


Fig.12: Symmetric PM4: Load displacement curve

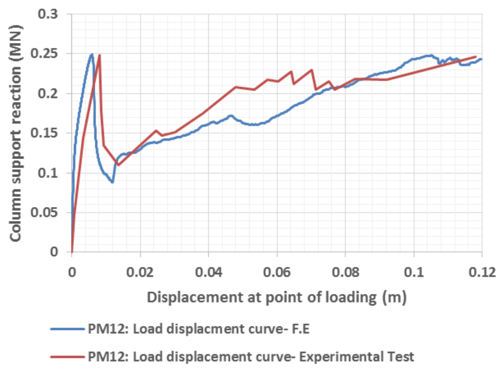


Fig.13: Symmetric PM12: Load displacement curve

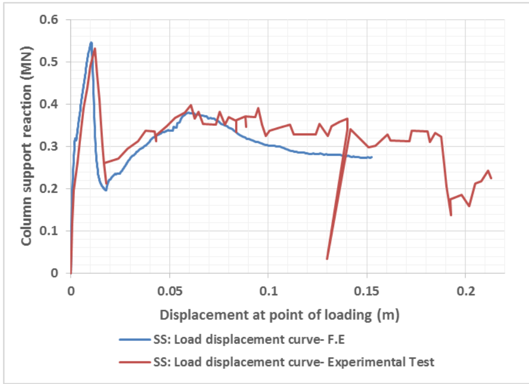


Fig.14: Symmetric SS: Load displacement curve

3.3 Asymmetric assessment

F.E asymmetric assessment of punching shear and post punching shear response were obtained and are as shown in figures 15 and 16. Punching shear strength values were found to be lower than those of symmetry models due to unbalanced moments at opposite faces of columns. This was as expected. Asymmetric FE models also gave post-punching shear strength values which were lower than those obtained in symmetric models. For the PM4, a peak post punching shear strength for asymmetric slab specimen was found to be 12 percent lower than that of the symmetric slab specimen which was not significant. However, PM12 which contained integrity reinforcement gave a peak post punching shear strength for asymmetric slab specimen which was found to be 30 percent lower than that observed for the symmetric specimen. Peak post punching shear strengths for PM12 were taken at a displacement of 0.12m. This observed difference for the PM12 specimen was quite significant.

Values of residual punching shear capacities in asymmetric models were higher than those of symmetric models. This is believed to be due to the early activation of flexural and integrity reinforcements at the critical side. The PM4 slab model experienced fracture of flexural reinforcement close to the column face at the critical side but PM12 experienced this at both the critical and adjacent sides. Enhanced activation of flexural reinforcements are believed to be responsible for the observed fracture of flexural reinforcement in asymmetric cases. Fracture of flexural reinforcements brought about drops in column support reactions at displacements 0.1m and 0.064m as observed in figures 15 and 16 respectively.

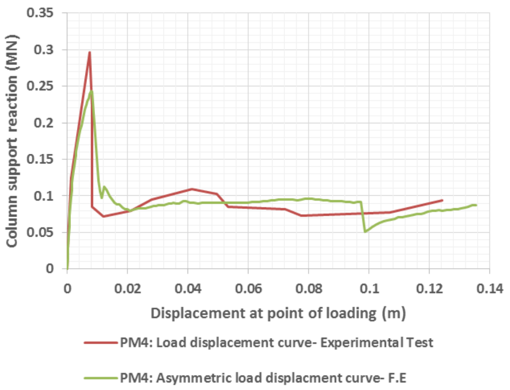


Fig.15: Asymmetric PM4: Load displacement curve

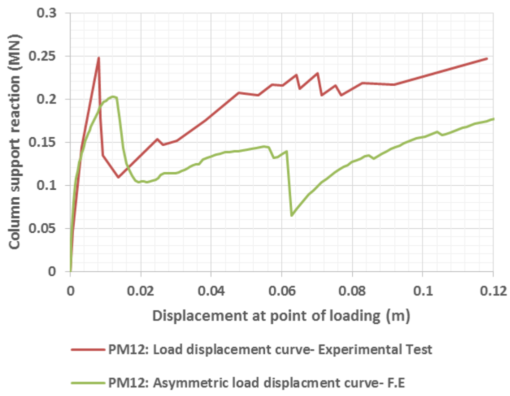


Fig.16: Asymmetric PM12: Load displacement curve

4 Discussion

Results of analyses showed 0.01m element length and 0.1667 e.d/s.d to be the ideal element characteristics for modelling for accurate modelling of punching shear, residual punching shear and post punching shear response of flat slab connections. Use of 0.02m element length would also give fairly accurate results if adopted to reduce computation time. With element size having no influence on residual punching shear and post punching shear strength, the use of elements of larger lengths could be used at a distance of about $3d_{eff.}$ from the column face without influencing the connection response. A reduction in post punching shear strength was observed due to asymmetric loading which could occur after the loss of an internal column. Without the consideration of such reductions in robustness assessment, the robustness of such structures could be significantly overestimated. This reduced connection capacity should also be taken into consideration during the design of integrity reinforcements for new flat slab structures. Results proof that accurate modelling of the response of flat slab connection could be carried out using a quasi-static analysis on LS-DYNA. Hence, this modelling approach could be incorporated in the system assessment of flat systems both in the quasi-static and dynamic domain.

5 Summary

Finite element modelling of the response of isolated flat slab connections were carried out using by means of quasi-static analyses using LS-DYNA finite element software. Sensitivity analyses showed that 0.01m element length and 0.1667 element depth to slab depth ratio gave the best predictions of slab response in punching shear strength, residual punching shear strength and post punching shear strength. Good prediction of these strength parameters were obtained for all slab test specimens assessed. In addition, assessment of the influence of asymmetry which could result from loss of an adjacent column showed a 12 percent decrease in peak post punching shear strength for slab specimen without integrity reinforcement but a 30 percent decrease for slab specimen with integrity reinforcement.

Acknowledgements

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