A comparison of damage and failure models for the failure prediction of dual-phase steels

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1 Introduction

The aim of this contribution is the comparison of different damage failure models that are available in LS-DYNA. In particular, the focus is concentrated on the failure prediction of dual-phase steels which are largely used in the automotive industry. Typically, such alloys provide a good compromise between ductility and strength for which this kind of material is also often used in safety relevant components. Examples are parts of B-pillars, side rails and cross members, i.e., parts that may be subjected to intensive loadings in a high speed car crash scenario. In contrast to some other usual alloys, dual-phase steels are often reasonably isotropic and well described by J2-based plasticity. This allows the use of simple and very efficient material formulations (e.g., *MAT_024 in LS-DYNA [1]) without excessively losing accuracy in crash simulations. Despite the fact that the elastoplastic behavior of such alloys can be generally well captured by simple plasticity models, the fracture behavior in practical applications still demands the consideration of several effects like stress state dependence, nonlinear paths, material instability, spurious mesh dependence, among others. Therefore, we consider three different damage/failure models available in LS-DYNA in order to calibrate the fracture behavior of a typical dual-phase steel: (a) the GISSMO damage/failure model [1–3]; (b) the Gurson-Tvergaard-Needleman constitutive model [4–8] and the Cockcroft-Latham fracture criterion as implemented in *MAT_135 in LS-DYNA [1, 9–11]. We shed some light on the differences among these models and verify their ability in reproducing experimental data on the coupon level for different stress states. The goal is to understand the advantages and limitations of each model concerning the prediction of failure. A detailed discussion then follows the results obtained with the three models.

2 Fracture and damage failure models

2.1 GISSMO damage/failure model

The Generalized Incremental Stress State Dependent Model (a.k.a. GISSMO) was conceived and developed by Neukamm et al. [2] and later formally presented in more detail by Andrade et al. in [3]. These contributions comprehensively describe the GISSMO model, therefore, we shall here only briefly present the important equations. A damage (D) and an instability measure (F) are simultaneously accumulated through

$$\dot{D} = \frac{n}{\Lambda(L_e,\eta)\varepsilon_f(\eta)} D^{\left(1-1/n\right)}\dot{\varepsilon}^p \tag{1}$$

$$\dot{F} = \frac{n}{\varepsilon_{crit}(\eta)} F^{\left(1-1/n\right)}\dot{\varepsilon}^p \tag{2}$$

where n is a damage exponent and ε^p is the accumulated plastic strain. $\varepsilon_{crit}(\eta)$ and $\varepsilon_f(\eta)$ are respectively the critical strain and the failure curve as a function of the stress triaxiality, $\eta = -p/q$. $\Lambda(L_e,\eta)$ is a function dependent on the element size and also on the triaxiality, given by

$$\Lambda(L_e,\eta)=\begin{cases}\beta_{shear} & \eta\le 0\\ \left\{\dfrac{\alpha(L_e)-\beta_{shear}}{1/3}\right\}\eta+\beta_{shear} & 0<\eta\le 1/3\\ \left\{\dfrac{\alpha(L_e)-\beta_{biaxial}}{1/3}\right\}\eta+\beta_{biaxial} & 1/3<\eta<2/3\\ \beta_{biaxial} & \eta\ge 2/3\end{cases} \tag{3}$$

Where $\alpha(L_e)$ is generally a monotonic decreasing function and the factors β_{shear} and $\beta_{biaxial}$ are defined as

$$\beta_{shear}=1-[1-\alpha(L_e)](1-k_{shear}) \tag{4}$$

$$\beta_{biaxial}=1-[1-\alpha(L_e)](1-k_{biaxial}) \tag{5}$$

The function $\Lambda(L_e,\eta)$ is necessary for the proper regularization of the damage/failure model in order to compensate for the effects of spurious mesh dependence. The factors k_{shear} and $k_{biaxial}$ can vary from 0.0 to 1.0.

Finally, damage/stress coupling is considered through

$$\boldsymbol{\sigma}=\left(1-\tilde{D}\right)\tilde{\boldsymbol{\sigma}} \tag{6}$$

$\tilde{D}$ is the damage that takes place when strain localization arises and is given by

$$\tilde{D}=\begin{cases}0, & if\ \ F<1\\ \left(\dfrac{D-D_{crit}}{1-D_{crit}}\right)^m, & if\ \ F=1\end{cases} \tag{7}$$

where D_{crit} is the accumulated damage when $F=1$ and m is the so-called fading exponent. At this point, it is important to emphasize that the functions $\varepsilon_{crit}(\eta)$, $\varepsilon_f(\eta)$ and $\alpha(L_e)$ are not predefined in the GISSMO model. This means that they can assume any expression. This ensures high flexibility when calibrating the failure model on experimental data.

The GISSMO damage/failure model is implemented in LS-DYNA in *MAT_ADD_EROSION and can be activated by setting IDAM=1. The modular nature of *MAT_ADD_EROSION allows GISSMO to be combined with any other plasticity model available in LS-DYNA. The functions $\varepsilon_{crit}(\eta)$, $\varepsilon_f(\eta)$ and $\alpha(L_e)$ are defined in the input as load curves through *DEFINE_CURVE.

2.2 Gurson-based model

We also present the constitutive equations based on the work of Gurson [4] and that are implemented in LS-DYNA in material models *MAT_120 and *MAT_120_JC. The following yield function is adopted:

$$\Phi=\left(\frac{\sigma_{eq}}{\sigma_y}\right)^2+2q_1 f^*\cosh\left(\frac{3q_2\sigma_m}{2\sigma_y}\right)-\left(q_1 f^*\right)^2-1\le 0 \tag{8}$$

where q_1 and q_2 are parameters introduced by Tvergaard [5] to better match experimental data. The parameter f^* is given by

$$f^* = \begin{cases} f & f \le f_c \\ f_c + \dfrac{1/q_1 - f_c}{f_f - f_c}\left(f - f_c\right) & f > f_c \end{cases} \tag{9}$$

where f is the void volume fraction, f_c is the critical void volume fraction (which triggers the mechanism associated with coalescence of voids) and f_f is the void volume fraction at complete failure. The use of f^* has been introduced by Tvergaard and Needleman [6] in order to capture the effects of coalescence of voids.

Based on the works by Gurson [4], Tvergaard and Needleman [6] and Nahshon and Hutchinson [7], the evolution of void volume fraction is given by

$$\dot{f} = \left(1 - f\right)\dot{\varepsilon}_v + A\dot{\varepsilon}_M^p + k_w f\omega\left(\xi\right)\frac{\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^p}{\sigma_{eq}} \tag{10}$$

where $\dot{\varepsilon}_v$ is the volumetric strain rate, k_w is a material parameter that controls the evolution of porosity under shear stress states (only available in *MAT_120_JC), $\dot{\varepsilon}_M^p$ is the equivalent plastic strain rate of the material matrix and A accounts for the nucleation of voids through

$$A = \frac{f_N}{s_N\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\varepsilon_M^p - \varepsilon_N}{s_N}\right)^2\right] \tag{11}$$

In the equation above, f_N is the void volume fraction of nucleating particles, ε_N is the mean nucleation strain and s_N denotes the standard deviation. The function $\omega\left(\xi\right)$ is defined as

$$\omega\left(\xi\right) = 1 - \xi^2 \tag{12}$$

where the normalized third deviatoric invariant, ξ, is given by

$$\xi = \frac{27}{2}\frac{J_3}{\sigma_{eq}^3} \tag{13}$$

In the constitutive models originally proposed by Gurson [4], Tvergaard and Needleman [6] and Nahshon and Hutchinson [7], no measure against spurious mesh dependence was generally taken, although some other contributions, either published by the same or other authors, proposed solutions to deal with the problem, for instance Feucht [8] or Tvergaard and Needleman [12].

In the case of LS-DYNA, there is the possibility of defining the variables f_0, f_N, f_c and f_f as element size dependent quantities in order to alleviate spurious mesh dependence when working with different mesh sizes.

2.3 Cockcroft-Latham fracture criterion

Cockcroft and Latham [9] proposed a fracture criterion based on the maximum principal stress where fracture is expected to occur if the following condition is fulfilled

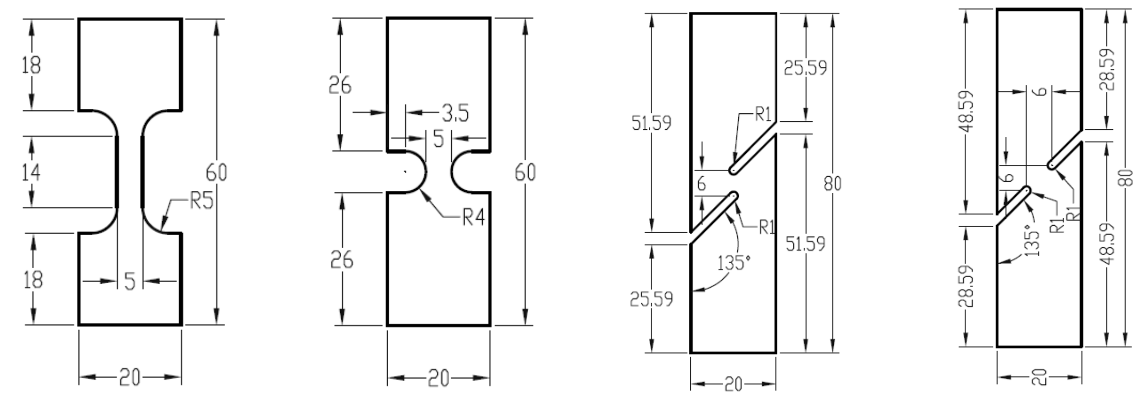
$$\int_0^{\varepsilon^f} \max\left(\sigma_1, 0\right) d\varepsilon^p = W_C \tag{14}$$

In the equation above, σ_1 is the maximum principal stress, ε^p is the accumulated plastic strain, ε^f is the strain at fracture and W_C is a critical value that has to be calibrated on experiments. In LS-DYNA, this criterion is currently (release R9) implemented in material models *MAT_107, *MAT_135 and *MAT_135_PLC. In this paper, we will adopt the implementation of *MAT_135. It is important to remark that the Cockcroft-Latham criterion is simply a fracture criterion independent of the plasticity model. This is in contrast with the GISSMO and Gurson-based models for which coupling between damage and plasticity generally occurs (in the case of GISSMO, this can be activated or not).

*MAT_135 does not directly offer the possibility of defining element size dependence. However, Wang et al. [10] and Lademo et al. [11] suggest the use of *MAT_NONLOCAL in combination with *MAT_135 in order to avoid spurious mesh dependence. The same authors suggest adopting the plastic thickness strain rate to be nonlocal in order to regularize thinning instabilities which would, in principle, eliminate the effects of spurious mesh dependence.

3 Calibration of a dual-phase steel

In this section, we calibrate the parameters of the three models based on the experimental data of a typical dual-phase steel. Physical tests with a set of different sample geometries based on the works of Sun et al. [13] and Basaran [14] were carried out in order to characterize the material (see Figures 1–3). All specimens were discretized with standard shell elements (EFORM=16 in LS-DYNA) using an element size of approximately 0.5mm in the critical zone.



(a) Small tensile (b) Notched tensile (c) Shear 0° (d) Shear 45°

Fig.1: Flat specimens used in the calibration procedure.

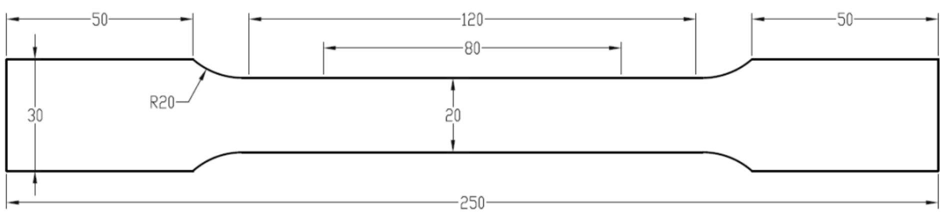


Fig.2: Large tensile specimen.

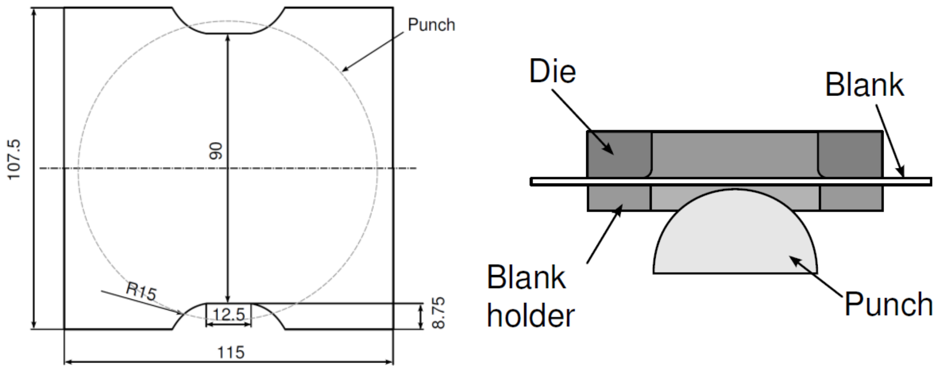


Fig.3: Biaxial specimen.

3.1 Calibration of the GISSMO damage/failure model

The calibration of the GISSMO damage/failure model including all its capabilities requires the identification of several parameters, including $\varepsilon_{crit}(\eta)$, $\varepsilon_f(\eta)$, $\alpha(L_e)$, n, m, k_{shear} and $k_{biaxial}$. However, not all these parameters must be necessarily calibrated. As a matter of fact, the user might decide, to some extent, which features of the GISSMO model he wants to consider. For instance, the definition of a critical plastic strain as a function of the triaxiality (ECRIT in LS-DYNA) is not mandatory. In case the user decides not to use it, it is advised to set DCRIT=1.0 in LS-DYNA to avoid premature damage/stress coupling. In this case, the GISSMO model would act simply as a fracture criterion without any influence on the material behavior itself.

In this study, the authors have decided to use the critical plastic strain because it allows the coupling between damage and stress under strain localization. Such coupling is interesting to better describe necking when working with shell elements which are larger than the actual fracture process zone. Also, the stress reduction caused by the damage/stress coupling in critical zones contributes to keep the critical behavior in concentrated spots in complex structures. Generally, this agrees better with experimental results of components made of ductile metals.

The fracture curve was calibrated by means of a reverse engineering strategy where the input curve was iteratively identified until a good agreement between simulation and experimental curve was achieved for all specimens considered (see Figures 4 – 5). A cubic spline interpolation was adopted and the points used for the interpolation are depicted in Figure 6. The same interpolation strategy was used for the calibration of the critical plastic strain curve where the strain at $\eta = 1/3$ is the true necking strain of the material. The damage and fading exponents were set to $n = 2.0$ and $m = 2.5$, respectively. The calibrated values of $\alpha(L_e)$, k_{shear} and $k_{biaxial}$ will be presented in the next section when the subject of spurious mesh dependence will be discussed.

3.2 Calibration of the Gurson-based model

The calibration procedure for the Gurson-based model followed a similar strategy suggested by Dunand and Mohr [15], see also Andrade et al. [3]. The calibrated parameters for the Gurson-based model are $f_0 = 1.0 \times 10^{-6}$, $\varepsilon_N = 0.127$, $s_N = 0.1$, $f_N = 0.01$, $f_c = 0.012$, $f_f = 0.03$, $k_w = 0.6$, $q_1 = 1.5$ and $q_2 = 0.7$.

3.3 Calibration of the Cockcroft-Latham fracture criterion

In comparison to the other two models, the calibration of the Cockcroft-Latham fracture criterion is very simple and only requires the identification of a single, critical value W_C. In this study, we iteratively calibrated W_C on the tensile test until the engineering strain at fracture matched the one of the experiment. For the present material and element size ($L_e = 0.5mm$), a value of $W_C = 0.55\ GPa$ was identified. *MAT_135 was used where the anisotropic coefficients $a_1 - a_8$ were all set to 1.0, i.e., it was assumed the material is isotropic. Furthermore, k was set to 1, which means that von Mises plasticity (like in *MAT_024) is recovered.

3.4 Discussion of the results

In Figure 4, the experimental as well as the simulation curves for the engineering stress strain curves are depicted for four of the specimens: small tensile, notched, shear 0° and shear 45° tests. Interestingly, the simulation curves deliver excellent agreement with the experiments when comparing the stress levels. This is an indicator that the J2 (von Mises) plasticity model adopted is quite able to reproduce the straining behavior of the material. In Andrade et al. [3], a comparison of the strain fields in the simulation and in the experiments obtained through digital image correlation supports this statement. The exception is the shear 45° test. In this case, the stress level in the simulation is higher than in experiment. As demonstrated in [3], there is also a mismatch in the strain field between simulation and experiment in the shear 45° test, indicating that the plastic straining of the particular specimen is not correctly captured in the simulation. Recent investigation carried out by the authors has shown that this effect is, to some extent, related to spurious deformation as a consequence of the use of a local model in the simulation. In this respect, the use of a nonlocal formulation (e.g., through *MAT_NONLOCAL in LS-DYNA) is able to significantly alleviate this effect and reduce the stress level similar to the one measured in the experiment. Nevertheless, the nonlocal method is restricted to the very small mesh sizes and, in the authors' opinion, not suitable for the large elements often used in car crash simulations. Also, due to the asymmetrical nature of specimen, lateral displacements in the clamping may occur, which also have an effect on the overall stress level.

Regarding the fracture, all models could reproduce the engineering strains at fracture with similar quality for the four planar specimens with exception of the shear 0° test for which the Cockcroft-Latham criterion predicted premature failure. These results are quite interesting, especially because the three models need a different number of parameters to be identified. In particular, the Cockcroft-Latham criterion can predict fracture with astonishing accuracy for different stress states by using only a single material parameter which, in turn, has to be calibrated by using experimental data of a single experiment. To some extent, these results corroborate some of the assumptions originally made by Cockcroft and Latham [9]. For instance, they argued that the prediction of fracture should include not only a strain but also a stress measure. Furthermore, they reasoned that the principal stress is a more appropriate measure than the equivalent stress because it can better describe the ductility of the material for different shapes of the necked region. In a perhaps more modern approach, we would today interpret this idea by saying that the triaxiality varies in the necked region and, because fracture is triaxiality-dependent, the ductility (or fracture behavior) of the material is different along the neck.

In a certain sense, the Cockcroft-Latham criterion can be interpreted as “energy” or “work” based, which is, to some extent, conceptually different from the underlying ideas of GISSMO. Also, the Cockcroft-Latham criterion predicts finite values for the shear fracture strain (in sharp contrast to the original model by Gurson).

Significant differences among the three models can, however, be observed in the simulation of the biaxal test (see Figure 5). In this case, only the GISSMO model was able to match the correct displacement at fracture. In fact, this was possible because the shape of the failure curve of GISSMO is arbitrary and, therefore, it can be adjusted to reproduce the experimental data with good accuracy. In the case of the Cockcroft-Latham criterion, the fracture strain under a biaxial stress state is inherently determined by the criterion which has been calibrated on the tensile test and cannot be *ad-hoc* modified. A better agreement would be possible by calibrating W_C on the biaxial test; however, in this case, the fracture of the other specimens cannot be reproduced anymore. In the case of the Gurson-based model, there is a certain flexibility in calibrating the biaxial fracture behavior by varying k_w. However, this parameter also affects the fracture strain under shear, which means that the simultaneous calibration of both specimens is, to some extent, limited.

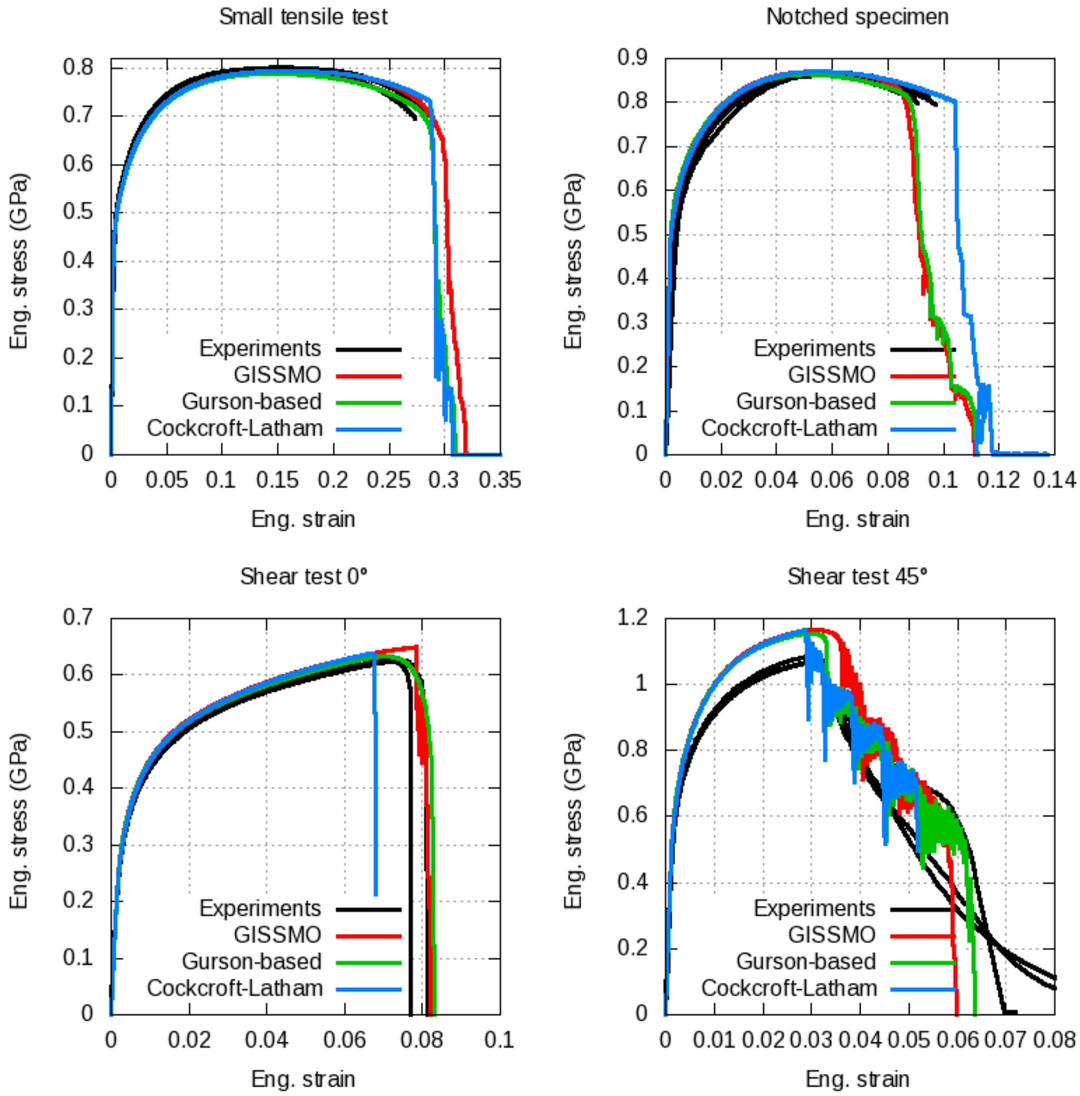


Fig.4: Comparison of experimental and simulation results for the flat specimens.

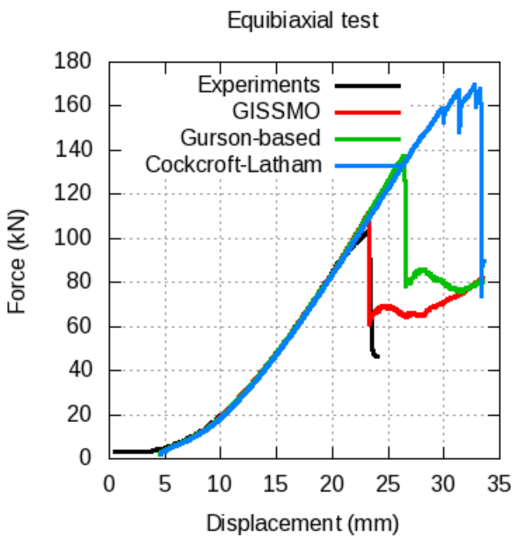


Fig.5: Comparison of experimental and simulation results for the biaxial specimen.

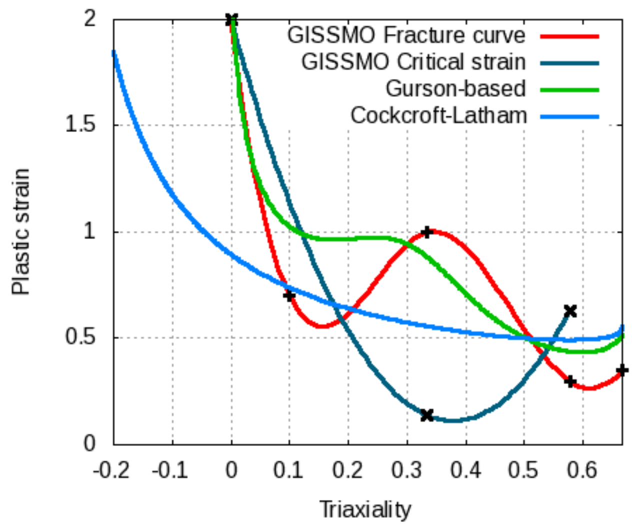


Fig.6: GISSMO fracture and critical plastic strain curves. Also depicted are the fracture curves for the Gurson-based model and the Cockcroft-Latham fracture criterion as a function of the triaxiality for the special case of linear strain paths (proportional loading).

Figure 6 shows the failure and critical plastic strain curves for the GISSMO model. Also depicted are the fracture curves for the Gurson-based model and for the Cockcroft-Latham criterion for the special case of linear strain paths, i.e., under proportional loading for which the triaxiality is constant from the virgin state of the material up to the fracture. The comparison of such curves should, however, be done with care because the different models accumulate damage in a different fashion. Nevertheless, in the case of the biaxial test, the triaxiality is quite constant, allowing a direct comparison of the fracture curves. In this case, it is possible to see that both the Cockcroft-Latham and Gurson-based model predict a late fracture because the fracture strain at triaxiality $\eta = 2/3$ is higher than in the case of GISSMO. The difference between the Gurson-based model and Cockcroft-Latham criterion can be explained by the nature of the underlying elastoplasticity models used. In the case of the Gurson-based model, plasticity is non-isochoric for which the effects of hydrostatic pressure affect the plastic straining with increasing damage. This leads to a certain softening behavior and also to a higher plastic straining in the fracture process zone where the failure strain is reached more rapidly.

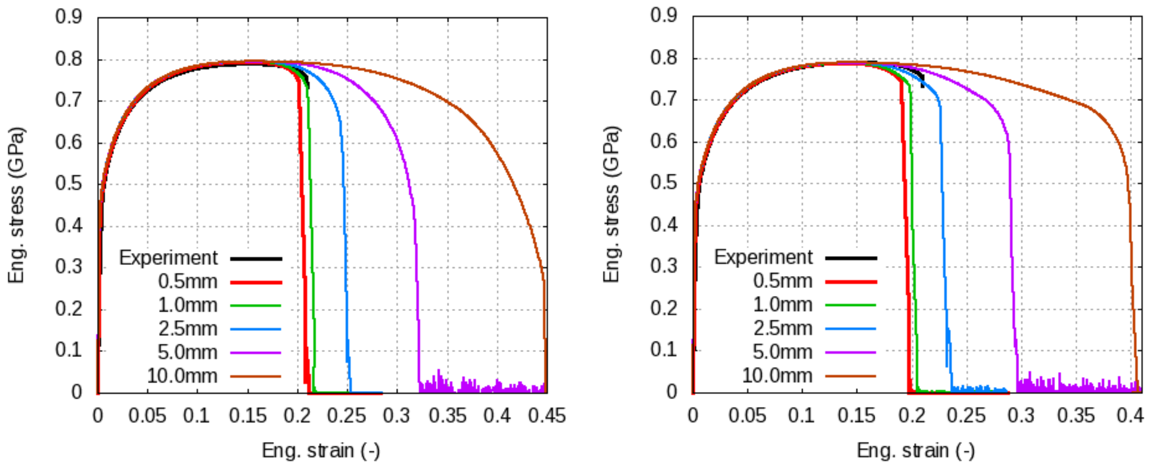
4 Mesh dependence and regularization

In this section, we simulate the large tensile specimen (Figure 2) discretized with element sizes 0.5mm, 1.0mm, 2.5mm, 5.0mm and 10.0mm. The aim is to observe the sensitivity of the numerical model with respect to the mesh size. As expected, if no regularization strategy is adopted, all models present spurious mesh dependence as can be seen in Figures 7 and 8.

In the case of GISSMO, the function $\alpha(L_e)$ (LCREGD in LS-DYNA) can be numerically calibrated in order to deal with the problem (see Figures 9 and 10). Although not investigated in detail in the present contribution, k_{shear} and $k_{biaxial}$ were set to 1.0 (see Andrade et al. [3]).

In the case of the Gurson-based model, the variables f_0, f_N, f_c and f_f can be defined as a function of the element size. However, in this particular study, only f_f was calibrated in order to compensate for the effects of spurious mesh dependence (see Figure 10). The results of the regularized material parameters are shown in Figure 9.

Finally, the Cockcroft-Latham criterion as implemented in *MAT_135 currently does not allow element size dependent parameters in the sense of GISSMO (through *MAT_ADD_EROSION) or of the Gurson-based models (*MAT_120 and *MAT_120_JC) in LS-DYNA. As suggested by Wang et al. [10] and Lademo et al. [11], the authors attempted to use the nonlocal criterion applied to the plastic thickness strain rate, unfortunately without success as the numerical solution still was mesh dependent. Nevertheless, further investigation is still needed to fully understand the effects of the nonlocal plastic thickness strain rate in this particular model.



(a) GISSMO (b) Gurson-based model

Fig.7: Simulation results of the large tensile specimen without regularization.

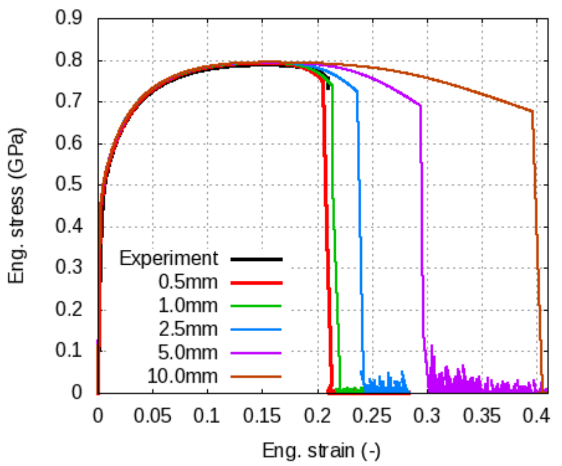
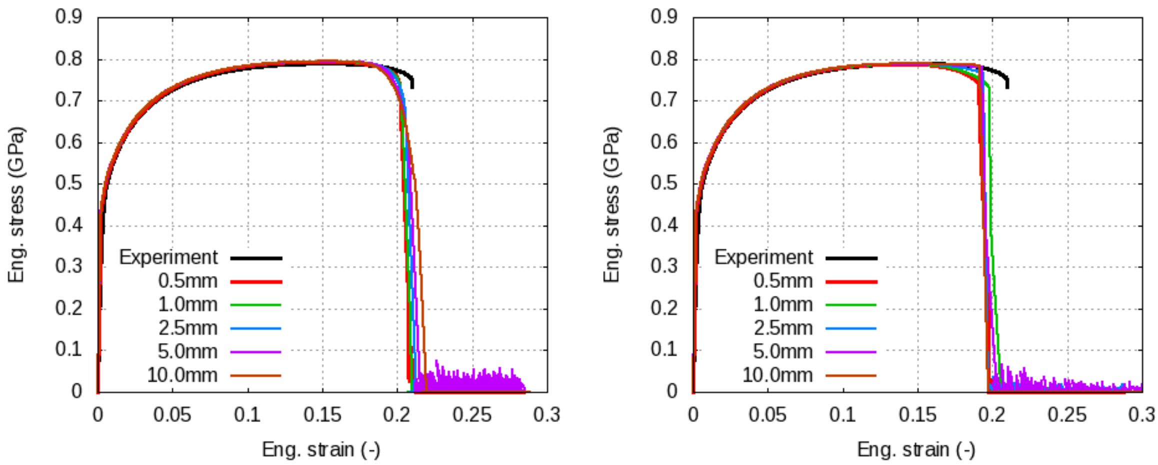
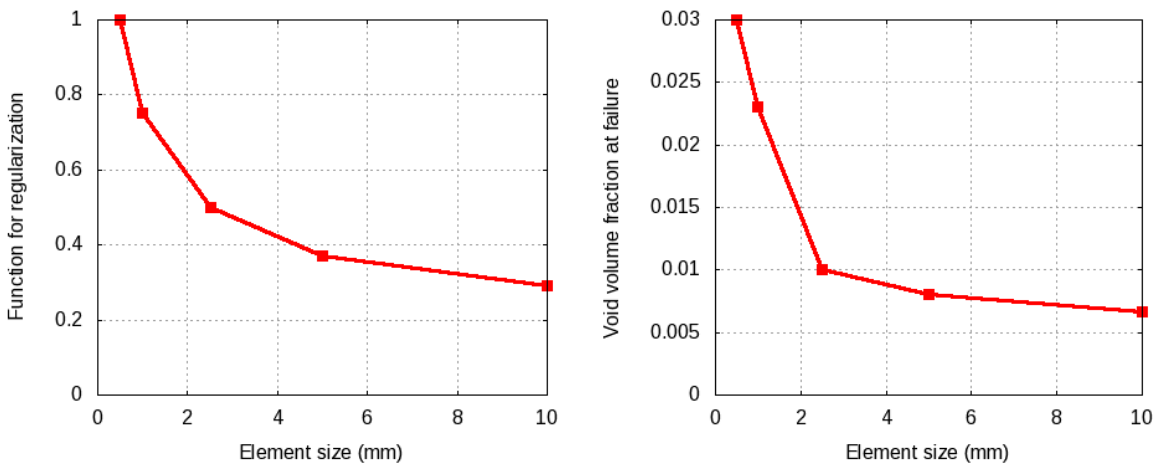


*Fig.8: Simulation results of the large tensile specimen without regularization for *MAT_135.*



(a) GISSMO (b) Gurson-based model

Fig.9: Simulation results of the large tensile specimen after regularization.



(a) GISSMO (LCREGD) *(b) Gurson-based model (LCFF)*

Fig.10: Calibrated regularization curves.

5 Summary

In this paper, we compared three different damage/failure models when subjected to the fracture prediction of a dual-phase steel alloy. On the coupon level, all three models showed, to some extent, similar quality for a constant element size (0.5mm) with exception of the biaxial test for which both the Gurson-based model and the Cockcroft-Latham criterion clearly underestimated the point of fracture. Whether this conclusion can be extended to other metallic alloys is questionable and more investigation is therefore needed.

Concerning mesh dependence, only the GISSMO (*MAT_ADD_EROSION) and the Gurson-based models (*MAT_120, *MAT_120_JC) could be regularized for larger element sizes. The Cockcroft-Latham fracture criterion as currently implemented in *MAT_135 presented high sensitivity with respect to the element size where the adoption of a nonlocal plastic thickness strain rate could not alleviate the problem.

Each of the models has its own advantages. For instance, GISSMO's high flexibility in the identification of material parameters (e.g., the fracture and critical strain curves can be arbitrarily defined) and its modular concept (can be combined with any plasticity model) are certainly attractive aspects for industrial applications like crash simulations. Gurson-based models are especially appealing due to their formulation based on micromechanics. However, some of the extensions made in the model in order to better match experimental data are purely phenomenological and, therefore, part of the physical motivation of the model is certainly lost. The Cockcroft-Latham criterion is interesting for its excellent ratio between simplicity and effectiveness as just a single parameter has to be identified. In the authors' opinion, the Cockcroft-Latham criterion can be an interesting choice when little experimental data is available (e.g., only the experimental curve of a tensile test). Nevertheless, the current lack of element size dependence in the implementation poses a considerable difficulty for using this model in practical applications.

It is also important to mention that certain crucial aspects have not been considered in the present study. For instance, the different criteria may significant differ under compression. This can be quite decisive in high speed crash applications where several components are subjected to highly compressive loadings. In the present contribution, no compression tests were performed, therefore, no comparison has been made in this sense. Furthermore, a comparison on component level would be essential to assess the ability of each of the different models in predicting failure in practical applications where this is left for future investigations.

6 Literature

[1] LTSC: “LS-DYNA Keyword User’s Manual – Volume II: Material Models”, Livermore, 2016.

[2] Neukamm F., Feucht M., Haufe A., Roll K.: “On closing the constitutive gap between forming and crash simulation”, 10th International LS-DYNA Users Conference, Detroit, 2008.

[3] Andrade F.X.C., Feucht M., Haufe A., Neukamm F.: “An incremental stress state dependent damage model for ductile failure prediction”, International Journal of Fracture, 200:127, 2016.

[4] Gurson A.L.: “Continuum Theory of Ductile Rupture by Void Nucleation and Growth: Part I—Yield Criteria and Flow Rules for Porous Ductile Media”. Journal of Engineering Materials and Technology 99(1), pp. 2–15, 1977.

[5] Tvergaard V.: “Influence of voids on shear band instabilities under plane strain conditions”. International Journal of Fracture 157, pp. 55–69, 1981.

[6] Tvergaard V., Needleman A.: “Analysis of cup-cone fracture in a round tensile bar”. Acta Metall 32, pp. 157–169, 1984.

[7] Nahshon K., Hutchinson J.W.: “Modification of the Gurson Model for shear failure”. European Journal of Mechanics A/Solids 27, pp. 1–17, 2008.

[8] Feucht F.: “Ein gradientenabhängiges Gursonmodell zur Beschreibung duktiler Schädigung mit Entfestigung”, PhD thesis, Technische Universität Darmstadt, 1999.

[9] Cockcroft M.G., Latham D.J.: “Ductility and workability of metals”. Journal of the Institute of Metals 96, pp. 33–39, 1968.

[10] Wang T., Hopperstad O.S., Lademo O.-G., Larsen P.K.: “Finite element analysis of welded beam-to-column joints in aluminium alloy EN AW 6082 T6”, Finite Element Analysis and Design 44, pp. 1-16, 2007.

[11] Lademo O.-G., Engler O., Keller S., Berstad T., Pedersen K.O., Hopperstad O.S.: “Identification and validation of constitutive model and fracture criterion for AlMgSi alloy with application to sheet forming”, Materials and Design 30, pp. 3005-3019, 2009.

[12] Tvergaard V., Needleman A.: “Effects of nonlocal damage in porous plastic solids”. International Journal of Solids and Structures 32, No.8/9, pp. 1063–1077, 1995.

[13] Sun D.-Z., Ockewitz A., Falkinger G., Andrieux F.: “Characterization and modeling of the deformation and damage behavior of thick-walled aluminum profiles for crash simulation”. International Congress on light materials: Science and Technology Light-MAT 2013, Frankfurt am Main, 2013.

[14] Basaran M.: “Stress state dependent damage modeling with a focus on the Lode angle influence”. PhD Thesis, RWTH Aachen, 2011.

[15] Dunand M., Mohr D.: “On the predictive capabilities of the shear modified Gurson and the modified Mohr-Coulomb fracture models over a wide range of stress triaxialities and Lode angles”. Journal of the Mechanics and Physics of Solids 59, pp. 1374–1394, 2011.